Mathematics Through Their Eyes:
Student Conceptions of Mathematics in Everyday Life

Fida Atallah

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ABSTRACT

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Fida Atallah, Ph. D.

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The study focuses on exploring student conceptions of mathematics and its utility in everyday life. The research is a survey with an iterative model consisting of eight stages. These stages were conducted through three inter-related studies: exploratory, pilot and final. I started with developing an open ended questionnaire for the exploratory study. Through data analysis, I derive categories of conceptions which I used to develop a draft questionnaire to be piloted. I introduced modifications based on the results of the exploratory and pilot studies. The final questionnaire, having both open ended and choice items, was conducted with a sample of 238 female students enrolled in an institute of higher learning in the Middle East.

The results of the final study indicate that around 86% the participants hold broad conceptions of mathematics as: (a) a school subject useful for everyday life, for work and for future studies (46%); (b) a form of mental activity useful for developing intellectual and problem solving abilities (31%) and (c) a group of numbers and rules for doing calculations (9%). These findings correspond with the three highest ranking conceptions of utility, which account for 68% of student responses. Students view mathematics as useful for: (a) day to day routines, work related tasks and academic and professional development (35%); (b) doing calculations and estimations (19%) and

developing intellectual skills (14%). The findings also correspond with the choice of intellectual skills and practical skills as the most useful. Together, these two skills account for 81% of the student responses, with intellectual skills coming first (58%).

Arithmetic was cited as the most useful subject in mathematics. It accounts for more than 80% of the responses. When asked to give examples of the use of mathematics for daily and work related tasks, several participants listed monetary applications, dealing with time, using numerical information and doing calculation as the most common.

The study, while exploratory in nature, provides some interesting and hopefully useful findings. As instructional designers, it is hoped that the results will guide us to make more informed assumptions and consequently more suitable decisions regarding the instructional design process by adding to our knowledge of learner characteristics and guiding us in selecting suitable instructional strategies and resources. For mathematics educators, its findings suggest the need for more dynamic classroom environments, for introducing a wide variety of meaningful contexts, for designing 'real life' learning activities, and for presenting students with different 'faces' of mathematics (e.g. cultural, civil, and aesthetic).

DEDICATION

I have learnt, early in life, to appreciate the value of learning and the important role that education plays in widening our horizon. I am dedicating this work to my mother Farida and father Hani for teaching me this valuable lesson and for encouraging me to pursue my dreams. I am also dedicating it to my brother Fadi for his constant support and encouragement.

I would also like to dedicate it to the special teachers whose dedication to their work encouraged me to think of teaching as a challenging and rewarding career.

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CHAPTER I

INTRODUCTION

> Mathematics is the key to opportunity. No longer just a language of science, mathematics now contributes in direct and fundamental ways to business, finance, health and defense. For students, it opens the doors to careers. For citizens, it enables informed decisions. For nations, it provides knowledge to compete in a technological economy. (National Research Council, 1989, p.1)

The twentieth century has brought about unprecedented innovations in many aspects of human civilization. With the start of the new millennium, we are witnessing dramatic changes in the way we live, work and learn. Our educational systems are struggling to cope with these changes, yet at times the pace is too quick. To be effective, changes in doing things require changes in the way we perceive them.

New philosophies of mathematics education are evolving in response to a rapidly changing world. Mathematics education is now viewed by some as a means to: "equip people with the knowledge and tools that will enable them to examine and criticize the economic, political, and social realities of their lives" (Zaslavsky, 1994, p. 217). Australian mathematics educator Nancy Shelly expresses strong views about the need for a different type of mathematics, a more humane mathematics aimed at benefiting people, not wars and profits. In a speech entitled *Mathematics: Beyond Good and Evil?* Shelly (as cited in Zaslavsky, p. 223) condemns the culture of militarism and proposes:

> ... the possibility of a different mathematics which grows out of a culture which has no pretensions to dominance: which can yet engage a learner creatively; which affirms humanness; and which attends human needs and will embrace earthly things.

Henderson (1981, p. 12) views the role of mathematics as a liberating power in people's lives:

> I believe that mathematics can be part of every person's understanding and can have an important role in the liberation of human beings. I define liberation as the removal of all barriers to a person's creativity.
>
> Specially, my assumption that *every person who needs some part of mathematics in order to understand some aspect of their experience can grasp that part of mathematics in a very short time.* All that is needed is confidence in their thinking and in their perception. This assumption applies, regardless of the person's mathematical background.
>
> ... when I listen to how other people view mathematics my understanding of mathematics changes. I am certain that as women, and members of the working class and other cultures participate more and more in the established mathematics, our societal conception of mathematics will change and our ways of perceiving our universe will expand. This will be liberating to all of us.

Bishop (1994) discusses cultural conflicts in mathematics education from the perspective of making mathematics accessible to everyone. He cites a *UNESCO* publication proposing that these conflicts concern several issues amongst which are values and beliefs. D'Ambrosio (1985) also addresses the issues of mathematics and culture in an article where he first coins the term *Ethnomathematics.* Ethnomathematics is concerned with the influence of socio-cultural factors on mathematics learning and teaching.

Another perspective on the importance of mathematics and its role in everyday life comes from the fact that an increased number of jobs require some form of mathematical skills. *The American Society for Training and Development (ASTD), The Conference Board of Canada* and other public and private sector organizations have launched major initiatives for identifying the basic skills needed for the modern workplace (see Carnevale et al., 1988; Carnevale, Gainer & Meltzer, 1990; The Conference Board of Canada, 1993). Mathematical skills are very much in demand. In *Everybody Counts,* a report by *the National Research Council* (1989) on the future of mathematics education, mathematics is considered to be the 'invisible culture'. It is often hidden from the public eyes, but its ideas are embedded in several aspects of our environment.

The terms *Numeracy* and *Quantitative Literacy* are often used in describing the basic mathematical skills needed in everyday life and for the workplace. In a statement published by the *US National Council of Supervisors of Mathematics (NCSM)*, numeracy was defined to include problem solving, applying mathematics to everyday situations, evaluating the reasonability of the results, using computational skills, estimation and approximation, geometry, measurement, reading, interpreting and constructing charts, graphs and tables, computer literacy and using mathematics for predictions (Galbraith, Carss, Grice, Endean and Warry, 1992). In *Everybody Counts*, numeracy implies the ability to deal with the mathematical demands of the adult life. Mathematics is viewed as involving more than calculations; "clarification of problem, deduction of consequences, formulating of alternatives and development of appropriate tools are as much part of modern mathematician's crafts as are solving equations or providing answers" (National Research Council, 1989, p. 5).

Quantitative literacy is used in Australia to represent one of three types of literacy: Prose, Document and Quantitative. It is defined as the ability to use mathematical operations to solve problems encountered in written material (Galbraith et al., 1992).

With the increased demand for mathematical skills, the level of mathematics possessed by some high school graduates is far from being adequate. *Everybody Counts* quotes Lester Thurow, dean of Sloan School of Management at Massachusetts Institute of Technology, as saying: "How can students compete in a mathematical society when they leave school knowing so little mathematics?" (p.1) Furthermore, mathematics often acts as a filter for scientific and professional jobs.

Everybody Counts cautions that when mathematics acts as a filter, it often filters students out of school (National Research Council, 1989). Ernest (1994, p. 213-214) criticizes the role of mathematics as a critical filter for entry into higher education, science and technology professions:

> A critique of the role of mathematics and mathematical knowledge in society might ask if there is a hidden agenda underlying the popular image of mathematics as difficult, cold, abstract, ultra-rational, important and largely masculine. Through such images it offers access most easily to those who feel a sense of ownership of mathematics, the associated values of western culture and the educational system in general. These will often tend to be middle-class, white and male. The image sustains their social privileges because mathematics acts as a critical filter for entry into higher education and professional occupations especially so where then sciences and technology are involved.

Student difficulties, underachievement, lack of motivation, and negative attitudes are well documented issues in the mathematics education literature. However, the literature on student conceptions of mathematics is not as comprehensive. I am interested in understanding how students perceive mathematics and its role in their everyday life. This interest stems from the belief that lack of perception of the links that exist between academic and everyday mathematics is partly responsible for the negative feelings towards mathematics, as well as the difficulties encountered in understanding mathematical concepts, and using them when needed.

Based on the above mentioned discussion, several questions come to mind. How can we develop an awareness of the role of mathematics in everyday life? How can we make use of the wealth of learning experiences that learners bring to their learning environments? How can we design learning environments that facilitate the thinking processes leading to the construction of mathematical knowledge? These are some of a long list of questions that we ponder about as we attempt to respond to the challenges facing us as educators and trainers. We need to explore learners' conceptions of mathematics and its perceived role in their everyday lives. We need to examine their use of mathematics in non-academic contexts. We need to understand learners' culture and the variables that influence their learning.

Understanding how mathematical knowledge develops involves understanding the culture in which it evolves. The next dialogue reprinted from *Through Indian Eyes: The Native Experience in Books for Children* edited by Slapin and Seal (as cited in Zaslavsky, 1994, p.166) illustrates the importance of the cultural context in mathematics education:

Teacher: All right, class, let's see who knows what two plus two is. Yes Doris?

Doris: I have a question. Two plus two what?

Teacher: Two plus two anything.

Doris: I do not understand.

Teacher: OK, Doris, I'll explain it to you. You have two apples and you get two more. How many do you have?

Doris: Where would I get two more?

Teacher: From a tree.

Doris: Why would I pick two apples if I already have two?

Teacher: Never mind you have two apples and someone gives you two more.

Doris: Why would someone give me two more, if she could give them to someone who's hungry?

Teacher: Doris, it's just an example.

Doris: An example of what?

Teacher: Let's try again - you have two apples and you find two more. Now how many do you have?

Doris: Who lost them?

Teacher: You have two plus two apples!!!! How many do you have altogether????

Doris: Well, if I ate one, and gave away the other three, I'd have none left, but I could always get some more if I got hungry from that tree you were talking about before.

Teacher: Doris, this is you last chance - you have two, uh, buffalo, and you get two more. Now how many do you have?

Doris: It depends. How many are cow and how many are bulls, and is any of the cows pregnant?

Teacher: It's hopeless! You Indians have absolutely no grasp of abstractions.

Doris: Huh!

In this dialogue, whether the child is not ready for the concept of addition or whether the teacher is using abstract language is a debatable issue. Introducing concrete examples such as apple, or culture specific ones such as buffalo, does not necessarily make the concept of addition more accessible to the child. What the teacher may have overlooked in this case was the situation that triggers the need for addition. Furthermore, this situation is very much related to the culture of that particular community of people. For example, sharing resources may be a matter of survival in the community hence the idea of keeping or taking more than what is needed may not be acceptable.

Research Goal

The research aims at exploring conceptions of mathematics and its utility in the eyes of its learners. A survey of student conceptions of mathematics and its applications was conducted using a researcher developed instrument.

Conceptions of Mathematics – A Definition

Schoenfeld (1985, p. 45) defines *belief systems* as "... one's mathematical world view, the perspective with which one approaches mathematics and mathematical tasks ". According to Schonefeld, belief dictates the focus and direction of a problem-solving attempt. Greene's (1971) belief systems, as summarized in Thompson (1992), and classified in terms of their relation with each other using three dimensions: (a) a belief is not held independent of other beliefs, some beliefs are related to each other as in primary and derivative beliefs; (b) beliefs are held with varying degrees of conviction, thus there are central and peripheral beliefs; and (c) beliefs are held in clusters, where these clusters are somewhat held in isolation from each other.

Andrews and Hatch (2000) propose that beliefs operate at two levels: (a) single beliefs and (b) clusters that form belief systems. Thompson (1992) suggests that belief systems serve as metaphors for examining and organizing individual beliefs, which are dynamic and change as a result an individual's experiences. She distinguishes between beliefs and knowledge and discusses two of the distinctive features of beliefs in relation to the research on teacher conceptions: (a) beliefs are held with varying degrees of conviction and (b) beliefs are not consensual.

In discussing Greene's views on belief systems, Pehkonen (2001, p. 18) presents four components that are part of an individual's mathematics related beliefs: (a) beliefs about mathematics, (b) beliefs about one's self as a learner and a user of mathematics, (c) beliefs about mathematics teaching, and (d) beliefs about mathematics learning. Pehkonen (p.15) views beliefs as "situated in the 'twilight zone' between the cognitive and the affective domain, since they have a component in both domains".

Although the distinction between beliefs and conceptions is not a "terribly important one", Thompson (1992, p. 130) considers conceptions to be "more general mental structures, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences and the like".

Osborne and Wittrock (in Mevarech and Kramarsky, 1997) describe student conceptions as categories of beliefs, theories, meaning and explanations. Oaks (1994, p. 43) characterized conceptions of mathematics as "the view students hold of mathematics, how they would describe mathematics, and what they believe is required in learning and doing mathematics".

In this study, the following definitions of conceptions of mathematics, sharing common features with Oaks's characterization, will be used:

Conceptions of mathematics are views that students hold of mathematics as a discipline i.e., what they believe it is and how they describe it.

Conceptions of the utility of mathematics in everyday life are views that students hold on the relevancy and utility of mathematics in the real world; how they describe its use in personal life and work situations.

Research Questions

The study aims at exploring student conceptions of mathematics in everyday life. In particular it aims at addressing the following questions:

Research Question One: What are student conceptions of mathematics as a discipline?

Is mathematics a set of rules and formulas for getting answers? Is it a symbolic reflection of what is out there? Is it a form of art studied for its aesthetic nature? Is it a logical system that promotes analytical skills? Is it a human endeavor aimed at helping people understand their world better? Is it a tool used for advancing other disciplines?

Research Question Two: What are student conceptions of the utility of mathematics in everyday life?

Why is mathematics useful? Is mathematics useful for solving everyday life problems? Is it useful for future studies? Is it useful in work contexts?

How is mathematics useful? What skills does it help us develop? Does it, for example, help us develop our thinking skills, our sense of creativity, and our scientific knowledge?

Which mathematical subjects and topics are useful and for what purposes?

Research Question Three: What are student conceptions of the links between mathematical knowledge and its applications in the work environment?

How do people in different professions use mathematics? What type of mathematics do they use?

CHAPTER II

THE LITERATURE

> One ... fact must astonish us, or rather would astonish us if we were not too much accustomed to it. How does it happen that there are people who do not understand mathematics? If the science invokes only the rules of logic, those accepted by all well-formed minds ... how does it happen that there are so many people who are entirely impervious to it (Poincare as cited in Sfard, 1991, p.1).

Knowledge accumulated through decades of research has not been able to adequately answer Poincare's question (Sfard, 1991). The difficulties encountered in learning mathematics exceed those faced learning other scientific disciplines. Sfard suggests that there is more to mathematics than rules of logic; the very nature of mathematical knowledge should be examined. The question to be asked is more qualitative than quantitative in nature - why mathematics is much less accessible than other disciplines.

A journey into the world of mathematics reveals a universe with its unique objects, such as numbers and sets (Sfard, 1991). However, advanced abstract mathematical concepts, such as functions, are not material objects. They are not accessible to the senses, and can be seen only with the minds. Thus, she concludes: "Being capable of somehow 'seeing' these invisible objects appears to be an essential component of mathematical ability; lack of this capacity may be one of the major reasons because of which mathematics appears practically impermeable to so many 'well-formed' minds' " (p.3).

According to Sfard (1991), mathematical concepts seem to have a dual nature – *structural* and *operational*. For example, a function is structurally defined as a set of ordered pairs. It may be viewed operationally as a method for calculating one system from another. She suggests that operational concepts precede structural concepts in the process of concept formation. In a later article Sfard (1994) discusses operational and structural modes of thinking in her research on mathematical understanding. She proposes to study mathematical understanding using the idea of *reification*. "Reification – a transition from an operational to a structural mode of thinking – is a basic phenomena in the formation of a mathematical concept" (p. 54). Illustrating Sfard's idea of reification, Hersh (1994) explains that in learning a new mathematical concept, children first learn it as an algorithm or procedure, which at a later stage changes into an entity or object. As an example, subtraction is an algorithm that reifies into negative numbers.

Sfard (1991) describes the term *concept* as a theoretical construct referring to a mathematical idea in its formal form. She uses the term *conception* to refer to "the whole cluster of internal representations and associations evoked by the concept" (p. 3). Da Ponte (in Andrews and Hatch, 2000) views conceptions as organizing frames of concepts, and consequently cognitive in nature. Steele (1994) describes conceptions as having two components: beliefs and knowledge, where belief can be both cognitive and affective. Ernest (in Steele, 1994) suggests that beliefs are personal views, assumptions and values.

Andrews and Hatch (2000) acknowledge that the literature on conceptions is not always clear. In addition, some writers prefer to use the word *perceptions* in place of conceptions. In this study, the term conception refers to views and shares similar features with the term *belief* as described by Ernest. However, it is used differently by different researchers cited throughout the study.

Capraro (2001) presents some views on conception and beliefs: Hersh views one's conceptions of mathematics as affecting the way it is presented; Corey views beliefs as the cornerstones "at the heart of our actions" (p. 4); Dewey views beliefs to be the best indicator of decisions that people make.

Mathematics Education: Study and Research

Mathematics as a Discipline

> Mathematics is as old as civilization itself. By the Neolithic Period, as life became settled and villages began to appear, writing and counting became increasingly useful. With counting the history of mathematics began. To count the passage of time, to weave intricate patterns in baskets or fabrics, and to apportion goods, crops, and livestock required a basic sense of arithmetic. ... The earliest knowledge of mathematics is preserved in Egyptian papyruses, Babylonian cuneiform tablets, and Greek manuscripts. They indicate that the first mathematical concerns involved arithmetic, algebra, geometry, and trigonometry (Grolier CD, 1997).

Mathematics has very ancient roots that date back to the dawn of human civilization. The early mathematical activities developed from primitive forms of counting or simple pictorial representations of objects. Throughout the earlier centuries of human civilization, mathematical developments were triggered largely by basic social and economical needs. Gradually, mathematics grew into a discipline having scientific, financial and statistical applications. Furthermore, mathematical thought was enriched by people who explored the philosophical and aesthetic aspects of mathematics. During the 19th and 20th century, " ... while mathematics continued to be applied to the standard problems of physics and astronomy, pure mathematics, divorced form physical problems, increasingly developed an impetus of its own" (Grolier CD, 1997).

At present, nearly all countries in the world are engaged in creating mathematics (Davis and Hersh, 1981). The mathematics of today, they continue, is unified, as opposed to the relative isolation in earlier times between eastern and western mathematics. Lancaster (1976) suggests that if the usefulness of mathematics did not extend beyond arithmetic, there would be no need for mathematics courses beyond middle school. However, he argues, the world that we live in is extremely complex, and mathematical applications are being used in a variety of domains such as space research, economic planning, statistics, computing science and others. In his book, *Mathematics Models of the Real World*, he introduces the concept of *mathematical model* – a unifying concept in applied mathematics. Applied mathematics is concerned with investigating real world phenomena – explaining and if possible predicting their future behavior. Lancaster cites examples such as the vibrations in the structure of an aircraft, the propagation of information through cells, the prediction of trends in populations and other applications. Davis and Hersh (1981) suggest that mathematical models can be used, among other things, to obtain answers about future events in the physical world, influence experimentations or observations and promote conceptual understanding and growth.

Research in Mathematics Education

Commenting on the nature and role of research in mathematics education, Silver and Kilpatrick (1994, p. 735) suggest that "... the field has a complex family tree. With roots in education and mathematics, and with long-standing ties to psychology and more recently developed ties to other scholarly disciplines, mathematics education is surprisingly difficult to characterize, and research in mathematics education is perhaps even more difficult to define".

In an article summarizing a quarter century of research on mathematics education, Kieran (1994, p 597) suggests that "we have traveled quite a distance from the days when, 25 years ago, understanding in mathematics education research was analyzed in terms of Bloom's taxonomy and measured by achievement scores or tests of immediate recall. In the process, we have come to realize that 'understanding is an ongoing activity' and that our research must reflect this stance". Sfard (1994) suggests that understanding involves going beyond the ability to solve problems and prove theorems. Understanding, she proposes, is both difficult to achieve and to explain.

There is growing interest in exploring mathematical understanding (Sfard, 1994; Pirie and Kieren, 1994), namely "understanding what understanding is about" (Sfard, p.44). In their theory about the growth of mathematical understanding, Pirie and Kieren suggest that understanding is "a constant, consistent organization of ones' knowledge structures: a dynamic process, not an acquisition of categories of knowing" (p.187). In her research on how mathematicians understand mathematics, Sfard (1994, p. 54) attempted to demonstrate that reification – a transition from the operational to the structural mode of thinking – is "... the birth of a metaphor that brings a mathematical object into existence... ", and helps us understand it better.

Our views of learning and understanding are evolving. Learning is now seen by some as a constructive activity – a process of creating meaning from experience. Constructivist researchers are interested in exploring the construction of mathematical concepts and the operations by which students "attend to and organize their experiences" (Steffe and Kieren, 1994, p.722).

Another perspective on mathematical learning is presented in Massingila's (1994) review of literature. Other than occurring in school settings, Saxe (1988) views mathematical understanding as also occurring in out of school contexts, when engaged in cultural practices. Schoenfeld (in Massingila, 1994) views learning and doing mathematics as an act of making sense and as having both cultural and cognitive aspects. This view is shared by Cobb who suggests that cognition is context bound. Cobb (1989) calls for analyzing the teaching and learning of mathematics using three distinct contexts: experiential, cognitive and anthropological. He proposes that "... mathematical meaning can be in the world (experiential), in the individual's head (cognitive), and in social interaction (anthropological)" (p. 41). He also suggests the need to complement cognitive constructivism with an anthropological perspective that emphasizes the role of cultural knowledge.

Prior to the past two or three decades, mathematics knowledge was considered to be 'culture free' (Massingila, 1994). More recently, the cultural history of mathematics has become a focus of interest. Massingila observes that this research examines mathematics in distinct cultures, as well as its use in everyday situations within cultures.

Examining mathematics practices in a whole culture is within the framework of ethnomathematics. D'Ambrosio (1985) attributes the previous lack of emphasis on ethnomathematical research to people's beliefs in the universality of mathematics. He reports on anthropological research that suggests the presence of typical mathematical practices such as counting, ordering, sorting, measuring and weighing that are done differently from the way they are taught in school mathematics. Bishop (1988) lists six fundamental universal mathematical activities that are necessary and sufficient for the development of mathematical knowledge. These activities are counting, locating, measuring, designing, playing and explaining.

Massingila (1993) discusses the research done on examining mathematics in distinct cultures as well as in everyday situations. She notes that some of this research has demonstrated the gap between mathematical practices in school and in everyday situations. Boaler (1993) marks an inconsistency in mathematical performance across school and everyday situations. Boaler also observes that recent research is challenging the belief " ... that mathematics can be learned in school, embedded within any particular learning structures, then lifted out of school to be applied to any situation in the real world" (p.12).

Gravemeijer (1994) observes that along with the changes in mathematics education, new educational research approaches are emerging. Traditional positivist paradigms are losing grounds, while interpretative ones are gaining momentum (Walker in Gravemeijer). There is need for studies that focus on what goes on in classrooms. Gravemeijer observes that the new trends in the field attempt to integrate research with principles of instructional design. This trend uses teaching experiments for curriculum development. This type of research lends itself to qualitative methodology where the emphasis is on understanding the situation rather than prediction.

Sierpinska, Kilpatrick, Balacheff, Howson, Sfrad and Steinbring (1993) suggest that mathematics education, and in particular research in mathematics education "lies at the crossroads of many well-established scientific fields such as mathematics, psychology, pedagogy, sociology, epistemology, cognitive science, semiotics, and economics" (p.276). Silver and Kilpatrick (1994) add other disciplines such as anthropology to the list. They emphasize the diversity of research methods and traditions governing the different disciplines, as well as the fact that mathematics education researchers rely on these older and more established disciplines to frame their work.

A controversial set of issues emerges along with the diverse approaches and traditions. Issues of respectability of the research and its practical impact on educational practices hold priority. These issues are not an exclusive concern of mathematics education research, but of educational research at large. Addressing educational psychologists, Fenstermacher and Richardson (1994, p. 53) present the following suggestion: "If the inclination is towards the discipline, that is where one finds one's problems – among the theories, studies, funded research, and questions that constitute the literature of the discipline. If the inclination is towards education, the problems are located in practice – that is, in actual attempts to instruct and learn".

Glesne and Peshkin (1992) suggest that when selecting the research methodology, we tend to select the one most consistent with our worldviews. It is not necessarily the research problem that determines the methodology; rather it is our own personal way of viewing and understanding the world. They cite Schwandt as saying (p. 9):

> Our constructions of the world, our values, and our ideas about how to inquire into those constructions, are mutually self-reinforcing. We conduct inquiry via a particular paradigm because it embodies assumptions about the world that we believe and the values that we hold, and because we hold those assumptions and values we conduct inquiry according to the precepts of that paradigm.

Qualitative research is gaining ground in the field of mathematics education. Well-known journals such as *The Journal for Research in Mathematics Education* are having an increasing number of published manuscripts based on qualitative methods. Lester (as cited in Silver and Kilpatrick, 1994) notes that while in 1973, nearly all the research reported in the journal was of the statistical hypothesis testing type, in 1993 only 37.5% were exclusively statistical. Silver and Kilpatrick observe that in recent years, qualitative research has emerged as the dominant type of research in mathematics education.

Qualitative research is a means to study mathematics in practice. Its value is not measured by impressive statistics about the number of published articles that utilize qualitative methods. Rather, it is reflected in its potential to provide insight into how people, as individuals and communities, perceive, learn and use mathematics.

Conceptions of Mathematics: An Overview

> For over two thousand years, mathematics has been dominated by an absolutist paradigm, which views it as a body of infallible and objective truth, far removed from the values of humanity. Currently, this is being challenged ... affirming that mathematic is fallible, changing and like any other body of knowledge, the product of human inventiveness. (Ernest as cited in Franke and Carey, 1997)

Thompson (1992) provides a brief history on beliefs and conceptions in an article synthesizing the research conducted in this area. Interest in the nature of beliefs and their influence on actions started in the early twentieth century in the field of social psychology. While this interest diminished for a few decades, it was renewed in the 1960s. In the 1980s interest in beliefs and belief systems had spread to a variety of disciplines. In education, Thompson attributes the interest in teacher beliefs and conceptions to paradigm shifts in the field. She summarizes the research interests as focusing on beliefs about mathematics, as well as about teaching and learning mathematics. The research done is mostly interpretive in nature. Some research was conducted with the purpose of describing or documenting beliefs and conceptions, while other research explored relations between conceptions and practices. Thompson views these beliefs as dynamic and changing as a result of experiences.

In her 1992 article, Thomson lists some major initiatives in the area of teacher conceptions and beliefs. Ernest discusses three conceptions of mathematics: *problem - solving*, *Platonist* and *instrumental* (also see Pehkonen, 2001; Benken & Wilson, 1996). The problem-solving view portrays mathematics as an expanding field of human invention – a 'process to be realized' – problem driven and dynamic. The Platonist view portrays mathematics as a 'formal system' – a discovered body of knowledge – a static product. The instrumental view portrays mathematics as a 'tool box' – facts, rules and skills to be used skillfully.

Corresponding to Ernest's view, Thompson presents Skemp's two types of conceptions: *instrumental* and *relational.* The instrumental view portrays knowledge as step by step procedures that students learn to do. The relational view portrays knowledge as knowing conceptual structures that enable students to construct plans to do their tasks.

Thompson also presents two conceptions identified by Lerman: *absolutist* and *fallibilist*. The absolutist view is that mathematics is based on universal and absolute foundations. The fallibilist view is that mathematics develops through conjectures, proofs and refutations – it involved an element of uncertainty.

In a study designed to explore the relation between teacher conceptions and their instructional practices, Thompson (1984) conducted a case study with three junior high school mathematics teachers. Three different views on mathematics emerged (p. 119): (a) mathematics is a "coherent subject consisting of logically inter-related topics" which emphasizes the meaning of the concepts and the logic of the procedures, (b) mathematics is a "challenging subject" which uses discovery and verification, and (c) mathematics is "prescriptive and deterministic in nature" .

Two of Thompson's subjects viewed mathematics as static body of knowledge – a finished product, while the third viewed it as dynamic. Thompson noted the differences in approaches to teaching and related them to the different conceptions the teachers hold.

Van Oers (2001) proposes several views on what constitutes real mathematics in classroom such as: (a) mathematics as a subject is about arithmetic operations, (b) mathematics as a subject is about structures, and (c) mathematics as a subject is a problem solving activity using symbolic tools.

The literature on student conceptions is not as extensive. In the next section, I will present some of the initiatives and findings.

Student Conceptions of Mathematics

> Probably no area of human activity is as afflicted as mathematics with a gap between public perceptions of its nature and what its practitioners believe it to be. (Barbeau as cited in Picker and Berry, 2001, p. 65)

Over the past few years, researchers have highlighted the importance of several affective variables such as emotions, attitudes and beliefs, in understanding student behavior in the mathematics classrooms (Gomez-Chacon, 2000). Gomez-Chacon attributes this interest in affective variables to several reasons: (a) their impact on how students learn and use mathematics, (b) their influence on a student's self concept as a learner, (c) the interactions produced between them and the cognitive system, (d) their influence on the social reality of the classroom, and (e) the obstacles that they may develop for effective learning.

Another area of interest is research on the images of mathematics and mathematicians. Furinghetti (1993) suggests that mathematics is a discipline for which each has a mental image regardless of their feelings towards it. Henrion (1997) proposes that imagery allows us to gain insight into people's underlying beliefs, assumptions and expectations. "Moreover, imagery not only reflects but affects what goes into mathematics and how mathematics is practiced (p. xix).

The *Curriculum and Evaluation Standards for School Mathematics*, known as *The Standards*, issued by *The National Council for Teachers of Mathematics* (NCTM) in 1989, suggests that for some students, their views of mathematics affect their learning (Buerk, 1994). These students view math as a discipline that requires rote learning. They view it as excluding their own thoughts and ideas. Buerk cautions that this rote view of mathematics is a "... self-perpetuating and addictive" view. It diminishes students' confidence in their ability to learn mathematics, and leads to passive behavior in the classroom. She emphasizes the need to see connections among students' thoughts, the concepts they study and their everyday surroundings.

Buerk (1994) suggests that misconceptions about learning mathematics are rooted deep in our society - parent, student, administrator expectation, as well as classroom experience. Different pressures created by curriculum content, higher standardized test scores, larger classes, and reduced staff may lead teachers to rush into finishing their programs. This means less time for thinking in class. Because of time constraints, teachers may start thinking for their students, thus encouraging student passive behavior.

Summarizing the wrong messages that students get in their mathematics classes, Bock (1994) presents a collection of myths that students are presented with: (a) mathematics is a set of rules, procedures, formulae, theorems, etc., (b) mathematics involves mostly calculations and equations that are either right or wrong, (c) mathematics has few applications in the real world, (d) all mathematics problems have answers and there is only one right answer, and (e) solutions to these problems usually require set procedures that students should learn from their teachers. Bock suggests that these myths lead to the development of perceptions that "obscure the beauty, the joy and the power of mathematics" (p.13). They result in students developing passive attitudes.

The view that student conceptions of mathematics affect the quality of their learning is also shared by several researchers (see Frid and White, 1995). According to Frid and White "... there are indications that student conceptions of mathematics affect the quality of related cognitive activities and learning outcomes ... how students interpret the content of their mathematics learning and hence how they relate to mathematics endeavors inside and outside school can influence mathematics performance" (p. 4).

In a study investigating secondary student and teacher conceptions of mathematics, their nature, purposes and outcomes, Frid and White (1995) report that interpretations of mathematics reflect personal and social factors. These factors are: (a) social status of mathematics, (b) utility of mathematics, (c) career aspirations and (d) interest or disinterest in mathematics. "The existence of these factors indicates that what it means to 'understand' mathematics is related to both a context and individuals' interpretations of a context" (p.1). They note that student conceptions of mathematics as well as their motivation for studying mathematics and their approaches to studying it have received little attention. Their study focuses on exploring the following (p. 2): What do students and teachers think mathematics is? What do students and teachers think are the intentions of mathematics study and why mathematics is included in school programs? What do teachers think that students think about mathematics and how do these views compare with those of students?

Frid and White's rationale for addressing these questions is their belief that mathematics education researchers need to see mathematics 'in others' eyes' to be able to understand its effects. Their statement: " ... until mathematics education researchers address and come to understand what mathematics is in others' eyes we are not in a position to understand the effects of mathematics courses" (p. 5), gave me the idea for naming my dissertation.

Frid and White (1995) suggest the need for answers to questions such as: What are student's personal views of the purposes of mathematics? Are these views stable or do they change over time? What is the effect of school on these views? What is the effect of school and outside school experiences on these views? What other factors may affect these views? Do curriculum developers, teachers, educational administrators, parents and general public views differ from students and how? Are educator and society expectations from school feasible in relation to mathematics education?

Reporting on the results of their study, the responses to the question of what mathematics is were found to be aligned with previous findings: "(1) numbers, rules and formulae, and (2) a logical processes or way of thinking. ... An additional conception of mathematics as a connected hierarchy that studies relationships or patterns was spoken of by some students, and these were generally students achieving at average or above average levels" (p. 6-7). As for the question regarding where mathematics came from, several students thought of mathematics as "... a human endeavor for addressing human needs and solving human problems" (p. 7). Students were somewhat undecided about whether mathematics was discovered or created. The researchers attribute the indecision to the following: students see mathematics as created in the sense that it is something that mathematicians made up, and that they (the students) do no see much use for or understand.

Grouws, Howald and Colangelo (1996) share the view that student conceptions of mathematics have been neglected. They suggest the need to seek answers to questions such as: Do students think that mathematics as a body of knowledge to be learned? Do they think that it is as a human activity involving exploration and discovery? Do they think that knowing mathematics means acquiring a collection of skills, formulas, and concepts that help solve problems quickly? Do some students think that mathematics involves exploring, observing patterns, representing information, generalizing, presenting logical arguments, etc? Do particular groups of students (e.g. secondary, gifted, at-risk ...) vary in their conceptions of mathematics, and why?

Mullis , Dossey, Owen and Phillips (as cited in Grouws et al., 1996) reporting on the results of a *National Assessment of Educational Programs* (NAEP) study on student perceptions, suggest that reform in mathematics requires helping students develop "... a lasting appreciation and positive attitude toward the use of mathematics to solve problems" (p. 3). Furthermore, they suggest the need for promoting an understanding of the power and utility of mathematics. The results of the study indicate that students with more positive attitudes and perceptions had higher proficiency in mathematics. Yet, these positive perceptions tend to diminish as they proceed to high school mathematics. The results of the study also support the link between student perceptions and learning.

Grouws et al. (1996) suggest the need for a deeper understanding of individual perceptions of mathematics as a discipline and the need to address questions such as: What does it mean to do or know mathematics? How do conceptions affect student use of knowledge and experiences, or their interpretation of learning situations?

In their study, designed with the purpose of conceptualizing a framework for analyzing student conceptions of mathematics, Grouws et al. (1996) explored both mathematically talented and high school algebra student conceptions, and how they relate to the nature of student learning. They identified the mathematically talented group based on multiple criteria including test results, teacher recommendations and student essays. These students were enrolled in a program for gifted and talented high school students. The high school algebra group was composed of intact classes enrolled in high school algebra courses. The framework they came up with, after reviewing the existing literature, developed from three major themes: (a) students' views of the nature of mathematical knowledge, (b) the character of mathematics activity and (c) the essence of learning mathematics. They developed three dimensions to characterize the first theme of student conceptions of the nature of mathematical knowledge: (a) composition of mathematical knowledge, (b) structure of mathematical knowledge and (c) status of mathematical knowledge. They developed two dimensions to characterize the second theme of student conceptions: (a) doing mathematics and (b) validating ideas in mathematics. The third theme was treated as a single dimension. An additional dimension, whose importance has been verified in several studies, was added – usefulness of mathematics. Each dimension had two poles. These dimensions and their poles were used to construct the *Conceptions of Mathematics Inventory*. The different themes and dimensions can be seen in Table 2.1 as presented by the researchers.

Table 2.1

The Dimensions of the Conceptions of Mathematics Inventory

I. Nature of Mathematical Knowledge	
Knowledge as concepts, principles, and algorithms	Knowledge as facts, formulas, and generalizations
Mathematics as a coherent system	Mathematics as a collection of isolated pieces
Mathematics as a dynamic field	Mathematics as a static entity
II. Nature of Mathematical Activity	
Mathematics as sense making	Mathematics as results
Logical thought	Outside authority
III. Learning Mathematics	
Learning as constructing and understanding	Learning as memorizing intact knowledge
VI. Usefulness of Mathematics	
Mathematics as a useful endeavor	Mathematics as a school subject with little value in everyday life or future work

The results indicate that mathematically talented students view mathematics as a dynamic coherent system of interrelated concepts. To them, mathematics is a process of sense – making guided by personal thought and reflection to establish the validity of mathematical knowledge.

Typical students also view mathematics as a dynamic system. However, they see it as a discrete system of facts and procedures that require memorization. They view doing mathematics as applying procedures and formulas previously learned and accepting them as true.

Regarding the last dimension, that of the usefulness of mathematics, which is a focus of the present study, both groups found that mathematics was useful in their personal lives as well as in the context of their future plans.

Oaks (1994) initiated a research project with the goal of describing the student world of mathematics from both cognitive and affective perspectives. The project helped in determining factors that influence student behavior towards mathematics – namely difficulties encountered in solving problems and inability to deal with mathematical theory. Oaks's study indicates that the conceptions students hold of mathematics affect their perception of class activity. Their expectation is that they will be provided with algorithms corresponding to different types of problems. Thus their rote conception of mathematics leaves no space for generating new methods for solving problems and makes them incapable of improving their performance. As a consequence, their grades do not improve and their frustration increases resulting in giving up on mathematics. Another important point mentioned by Oaks is that such students are unable to understand the logic and reasoning behind mathematics.

Gibson (1994) suggests using metaphors - comparing mathematics to specific objects to help students express their views of the subject and of themselves as learners. These mathematics metaphors allow teachers to know their students' conceptions of mathematics. According to Gibson (1994), passive attitude and behavior in mathematics classes should be a cause for concern. She remarks that in such cases the prevailing attitude amongst students is what she labels 'fast food' approach or "Give me the formula and some numbers, and leave me alone" (p.7).

Gibson (1994) used a written exercise provided by a colleague for examining student views of mathematics (see Table 2.2). I have used a modified version of one component of this exercise in my questionnaires.

Gibson asked students to describe mathematics using metaphors. Focusing first on the negative aspects of student response, she found out that many view mathematics as something beyond their control. Some held intense feelings such as anger, despair and frustration. Gibson also noted positive aspects; she noted that some students expressed feelings of challenge and excitement, as well as feelings of exhilaration.

Table 2.2

Mathematical metaphors exercise

Exercise Instructions	Time
List all the words or phrases you would use to describe mathematics to a friend.	wait 5-7 minutes
Imagine yourself doing or using mathematics either in or out of school; list all the feelings that come to mind.	wait 5-7 minutes
List all the objects (nouns, things) that you think math is like.	wait 5-7 minutes
Read over your three lists carefully (allow a lengthy pause here). From the third list, choose the word that best describes what math is most like for you. Finish this sentence: "For me, math is most like a (n) ______." Explain your choice fully in a paragraph.	allow 15-20 minutes

Note. This exercise is found in Gibson's article

Franke and Carey (1993) report that students (in this case – young children) view mathematics as a static body of knowledge that is replicated rather than created. Koch and Smith (1993), reporting their findings on minority student beliefs about what mathematics is to them, came up with six views: (a) memorized rules; (b) always forgotten; (c) irritating; (d) useful in the future; (e) needing insight; and (f) challenging.

Frank (in Pehkonen, 2001) derived five student beliefs from her research study: (a) mathematics is computation, (b) mathematics problems should be solved quickly in a few steps, (c) the goal of doing mathematics is getting the 'right answers', (d) the role of the mathematics students is to receive mathematical knowledge and to apply it, and (e) the role of the mathematics teacher is to transmit mathematical knowledge and to make sure that students have received it.

In a study focusing on adults with college or university level of education, Karsenty (2000) describes patterns of general opinions towards mathematics, two of which are relevant to the discussion on student conceptions. Some found mathematics to be an essential part of life that should be compulsory for all students. It is an important language of communication. It is needed for functioning in society. Some found it to be a powerful discipline. It is a tool for developing a rational personality.

Mathematics in Everyday Life and in the Workplace

Mathematics education is one of the areas that are hard hit by problems of low achievement, lack of motivation and low transfer of skills. Schools are being criticized for failing to produce graduates who have the required standards of mathematical knowledge that allow them to participate in work training programs, or enroll in professional ones. Even more acute are the claims that they are not producing graduates who have a reasonable level of numeracy that allows them to use their mathematical knowledge in everyday life. Furthermore, difficult challenges face mathematics educators in light of changing views on learning and teaching mathematics. Other than cognitive and affective aspects, there is a need to consider societal aspects that impact the field of mathematics education. Many questions await answers that may be elusive.

People need mathematics in their everyday lives – this is a self-evident fact. Yet, how much mathematics do they need? What mathematics do they need? Some advocate mathematics aimed at promoting social change through creating understanding and awareness of the role that mathematics plays in individual and social development. Some are concerned with mathematics as a discipline and maintaining its academic rigor. Others call for a mathematics that provides the foundations for workplace numeracy. These views suggest questions such as: Should we teach mathematics for its utility – to provide learners with the ability to use the mathematical skills in solving everyday problems? Should we teach it for its culture – to introduce learners to the culture of mathematics and its relation to the broader human culture? Should we teach it for the logic – to help learners to develop critical thinking and analytical skills? Should we teach it for employability – to help students develop one of the cornerstones of workplace basic skills? Should we teach it as a vital part in the chain of human knowledge, to provide a prerequisite for other disciplines?

These are amongst the goals advocated by different sectors of society with each championing its own cause. Amongst the many voices discussing the present and future of mathematics education, one voice receives little attention, that of the learners themselves. Although teachers often address student concerns through attempting to answer their typical question of why we need to study mathematics, they sometimes find themselves at loss for plausible answers. Most mathematics teachers find themselves having to confront this question and some try to address it (Karsenty, 2000). Karsenty relates this to "the gap between declarations and adult life reality" (p. 127). On the one hand there are all the typical arguments for studying mathematics to be able to cope with a rapidly changing world. On the other hand there is the skepticism regarding the relevance of several high school mathematics topics. Burke (as cited in Karsenty) expresses similar views: "Mathematics appears to be everywhere, but it is usually in the form of arithmetic or statistics – not abstract algebra or triangles. ... ... It is time to call a halt to specialized high school math requirements and not take math teachers word for what the rest of us need" (p. 127). Karsenty views that there is a shared element amongst the two views, that of valuable knowledge. One side sees this knowledge as useful, while the other side sees it as lifelong enrichment.

Davis (1995) lists some of the classical answers provided for why we need to study mathematics: it is needed for most jobs, for other disciplines, for fulfilling university requirements and for helping people think clearly. However, his interpretation of need is not typical: "We need to study mathematics to begin to understand our prejudices and to explore other possibilities for acting" (p. 6). He continues: "...learning mathematics affects who we are, what we do, how we stand in relationship to others, and how we situate ourselves in the world" (p. 8). In a later article, Davis (2001) revisits the question, asking this time why mathematics should be taught to all students. He suggests the need to ask what to teach rather than why – i.e. from providing a rationale for teaching mathematics to examining the topics of instruction.

A serious problem facing mathematics educators is the lack of perception of the link between mathematics and everyday life practices. Mathematics is seen as a form of abstract notions presented in an artificial manner. Mathematical problems are rarely accompanied by realistic contexts. This, among other things, results in problems of *inert knowledge* thus affecting the student's potential to transfer these mathematical ideas into everyday practices. The problem of inert knowledge as suggested by Whitehead in 1929 is represented by the distinction made between acquiring concepts and developing useful knowledge (Browns, Collins and Duguid, 1989). *The Cognition and Technology Group at Vanderbilt* (1990) also addressed this problem. They described inert knowledge as one that is not used spontaneously even though it is relevant.

Educators are concerned with the inability of high school graduates and adults to use school mathematics in real life contexts (see Boaler, 1998, Nyabanyaba, 1999). According to Boaler, many research projects have demonstrated that people tend not to use school-learned procedures in real life situations. Lave's work demonstrated that adults do not see the link between the two; they choose a procedure based on context rather than on the mathematics needed for the task. Thus, Boaler continues, Lave used the term *situated learning* to describe how learning is related to the situation in which it takes place.

Brown, Collins and Duguid (1989), in an article on situated cognition, caution that many educational methods ignore the context in which learning takes place. Namely, they are of the view that "... by ignoring the situated nature of cognition, education defeats its own goal of providing useable, robust knowledge" (p. 32).

In discussing mathematics education in the Netherlands, Wubbels, Krothagen and Broekman (1997) report on the *realistic* approach characterized by a shift from mathematics as created by others to mathematics as a subject to be created. This view distinguishes between concrete and realistic problems in the mathematics. For example, finding that the medians of a triangle are concurrent is a concrete problem, but not a realistic one. In addition, a problem may be contextual without being realistic. They propose that a problem is realistic only when experienced by the learner as real and interesting. They clarify that real does not always mean originating from students' everyday life. Such problems often arise when students work with manipulative material.

Sierpinska (1995) suggests a middle ground between the call for "learning mathematics in real-life contexts" and "learning of pure mathematics". She argues for the "learning of mathematics with applications". This stems from the belief that the ability to apply mathematics does not necessarily follow from the knowledge of mathematical theory. Students need to be taught the art of applications along with the mathematical knowledge.

In an series of studies on the mathematical components of professional expertise, conducted in the middle 1990's, Noss (2001) and his colleagues attempted to find "some mechanism to replace 'transfer' as the primary metaphor for the ways that mathematical knowledge is used in general" (p. 32). He reports that these studies indicate "... a disjunction between visible mathematics and what happens in practice". This means that the strategies used, in this case by nurses, did not have much to do with the numerical procedures taught. Noss refers to these strategies as the *artifacts of the practice*. These findings triggered the researchers' interest in characterizing professionals' mathematical knowledge, as well as how mathematics is used in work place situations.

In a later study, Noss, Hoyles and Pozzi (2002) voice concerns regarding the situated nature of knowledge in the work activities. They are of the view that situated cognition has created a *cul-de-sac* for mathematical learning. They suggest instead the notion of *situated abstraction*. In their own words: "Our notion of situated abstraction seeks to describe how a conceptualization of mathematical knowledge can be both situated and abstract. Mathematical conceptualization may be finely tuned to its constructive genesis – how it is learned, how it is discussed and communicated – and to its use in a cultural practice, yet simultaneously can retain mathematical invariants abstracted within the community of practice" (p. 205).

CHAPTER III

QUESTIONNAIRE DEVELOPMENT PROCEDURES

Research Environment and Participants

The research was carried out in a public institute for higher learning in the Middle East. The institute has several colleges at different geographic locations. The colleges offer a variety of technical and professional programs that prepare graduates for careers in government and private business sectors. The colleges also provide special customer – based training programs.

Admission to the colleges is restricted to nationals. To be considered for admission, a student must have a high school certificate. Mature students may be considered for admission based on their background and experience. In their first year, students are required to take a basic mathematics course which covers a variety of pre-requisite skills needed in their chosen fields. They are also required to take intensive English courses, as the language of instruction in most programs is English.

All the research studies, except for the formative evaluation done in the exploratory study, were conducted with participants enrolled in the colleges, as well as college faculty and staff. In all, three studies were conducted with 438 participants. During the exploratory study, the sample consisted of 184 participants both male and female. In the pilot study, the sample consisted of 16 participants both male and female. In the final study, there were 238 female participants.

Research Model

The research model consists of eight stages. It is iterative in nature with three interrelated studies: exploratory, pilot and final. In the exploratory study, I developed an open-ended questionnaire addressing the three research questions. Using the participant responses, I derived categories of conceptions for each questionnaire item. Next, I used these categories to construct the draft of a final questionnaire which would serve as the data collection instrument for the final study.

Following the pilot study, I introduced modifications to the questionnaire and conducted a formative evaluation of the modified questionnaire. Finally, I used the results of the previous stages to create the final version of the questionnaire to be used in the final study.

Table 3.1 summarizes the research stages involved in the three studies and provides a brief description of the function of each study and stage, as well as the instruments used.

Table 3.1

<u>Research studies and stages</u>

Research Study	Stage	Description
The exploratory study provides categories of responses that reflect learner conceptions of mathematics and its applications. EX-Q items are constructed using the three research questions.	1	Development and administration of the exploratory questionnaire EX-Q
	2	Content analysis and derivation of categories of responses to EX-Q
The pilot study provides a tested draft of the final questionnaire that addresses the research questions. IN-Q items are constructed using the categories derived in Stages 1 and 2.	3	Development and administration of the initial questionnaire IN-Q
	4	Response analysis and suggestions for modification to IN-Q
The final study provides descriptions of the student conception of mathematics and its applications as specified in the research questions. FN-Q items are modified based on the findings of Stages 1 thorough 4.	5	Development and administration of the final questionnaire FN-Q
	6	FN-Q response analysis
	7	Post Analysis: grouping FN-Q data by Age, Academic Level and Work Experience
	8	Summarizing the results of FN-Q

Research Instruments

I developed three questionnaires EX-Q, IN-Q and FN-Q for use in the three studies. They are described in Table 3.2. For complete versions of the questionnaires see Appendices A, B and C. The evolution of the items with the different questionnaires is illustrated in Appendix D.

Table 3.2

Research instruments

Instrument	Study	Description and Purpose
EX-Q	Exploratory Study	This questionnaire included open ended items that address the three research questions. It was used to derive categories of learner conceptions.
IN-Q	Pilot Study	This questionnaire included open ended as well as multiple choice items. It also included five short answer questions for formative evaluation purposes. It was piloted as the draft of the final questionnaire.
FN-Q	Final Study	This questionnaire included open ended, multiple choice and checklist items. It was used to gather data for the final analysis of conceptions.

Research Procedures

The Exploratory Stage

EX-Q Development

To examine the three research questions, I developed an exploratory questionnaire EX-Q with eight open ended items, referred to as EX-Q1 to EX-Q8. I provided an analogy with language to clarify some questions. Appendix D demonstrates the correspondence between the three research questions and EX-Q items.

I conducted a formative evaluation of EX-Q to check the clarity, language, layout and timing. I started with one to one interviews followed by a small group administration. The participants were graduate students, as well as faculty and staff in institutes of higher learning. Next I asked two English language teachers at the colleges where the research is taking place to check the language. Finally I discussed the questionnaire with two college administrators. Based on the results of the formative evaluation, I introduced language modifications as well as changes in the writing style. One major modification in writing style was using an objective form in an attempt to avoid leading questions. The following example serves to illustrate:

Original Item: What lessons did you find useful in mathematics?

Modified Item: Can you think of lessons that you found useful in mathematics?
If so, what were those lessons about?

The administrators strongly recommended that the questionnaire be translated to Arabic to make it easier for the students to express themselves. The final form of EX-Q was given to a certified translation office to produce an Arabic version.

EX-Q Administration and Analysis

I sent 270 copies to two colleges. The questionnaires were distributed by the mathematics teachers and collected during the same week. The rate of return was 184 out of 270, around 68%. Of the 184 participants, 37.5% were male and 62.5% were female. The breakdown of participants is shown in Table 3.3.

Table 3.3

<u>Breakdown of participants in Exploratory Study</u>

	Male	Female	Total
Students	67	108	175
Faculty and Staff	2	7	9
Total	69	115	184

The students were enrolled in diploma programs. Their ages ranged from 17 to 38 years old. Around 18% of the sample had previous work experience or were already holding jobs.

I checked the content for recurring themes, which were later grouped in categories. I applied the following procedure: (a) I performed the data entry using MS-Word, (b) I examined the recurring themes per question through a keyword search using the word processing feature EDIT-FIND, (c) Upon completion, I repeatedly checked the themes to derive categories and subcategories of responses, and (d) To verify the categories, I went back to the participant responses and labeled them based on the categories derived. Some responses belonged to more than one category; others could not be labeled because they were incomplete.

Analysis of Responses to EX-Q items

In the remainder of this section, I will discuss the content analysis for each of the eight questionnaire items.

<u>EX-Q1 and EX-Q2 Analysis</u>

> EX-Q1: If we were to describe language, we could say that it is a tool of communication, a means of expressing ourselves, etc. If you were to describe mathematics, what would you say?
>
> EX-Q2: We find language useful because it helps know and understand others better. It also helps us express our thoughts. Do you find mathematics useful? Why?

The responses to these questions were of a subjective nature. This necessitated a verification of the categories to ensure that they have captured the variety of themes reflected in the responses. The responses to EX-Q1 reflected two components – the 'essence of mathematics' and 'mathematics as a tool'. Consequently, the responses to EX-Q1 generated two sets of categories of conceptions: EX-Q1A (essence of mathematics) and EX-Q1B (mathematics as a tool).

Two raters, both graduate students of Education, were asked to individually read the participant responses and label them as belonging to one or more of the categories derived. I gave each rater a copy of the categories and their descriptions. The raters were instructed to place a 0 (ZERO) when unable to find a suitable category and to ignore missing or incomplete responses.

I used a Chi-Squared test to check the inter-rater reliability for the categories of conceptions EX-Q1 (essence of mathematics), EX-Q1B (mathematics as a tool) and EX-Q2 (uses of mathematics). The results indicated a significant difference between the matched and non-matched responses, in favor of the matched response in all cases (see Tables 3.4.1, 3.4.2 and 3.4.3 for details). For purposes of testing, the labeling per item was considered matched when raters chose the same category for a given item. For example, if the first rater labeled a participant response as belonging to categories 1 and 2, and the second rater chose exactly the same categories, this would be considered as a matched item. However, if the second rater chose only one category, or one or more different categories, this would be considered as a non-matched item.

Table 3.4.1

Chi-Squared test for EX-Q1A (essence of mathematics)

	Matched items	Non-matched items
Observed Frequency	137	37
Expected Frequency	87	87
Chi-Squared Value*	57.4713	

* Critical value as calculated by EXCEL function CHIINV = 6.6349 for one tailed probability p= 0.01 with df = 1

Table 3.4.2

Chi-Squared test for EX-Q1B (mathematics as a tool)

	Matched Responses	Non-matched Responses
Observed Frequency	145	29
Expected Frequency	87	87
Chi-Squared Value *	77.3333	

* Critical value as calculated by EXCEL function CHIINV = 6.6349 for one tailed probability p= 0.01 with df = 1

Table 3.4.3

Chi-Squared test for EX-Q2 (uses of mathematics)

	Matched Responses	Non-matched Responses
Observed Frequency	145	36
Expected Frequency	90.5	90.5
Chi-Squared Value *	65.6409	

* Critical value as calculated by EXCEL function CHIINV = 6.6349 for one tailed probability p= 0.01 with df = 1

Next, I set up a discussion session with both raters. I explained the purpose of the meeting as refining the categories though introducing possible conceptual or language modifications. The following cases where discussed: missing categorizations accidentally left out by the raters, cases when the raters were unable to categorize the responses and cases when the categories were unmatched. I provided clarifications to some questions pertaining to the category descriptions when requested. The raters were then instructed to re-examine the responses and re-categorize them if they deem it necessary. Table 3.5 presents the number of matched items before (Original) and after (Modified) the session.

Table 3.5

Breakdown of matched and unmatched responses before and after discussion

	EX- Q1A		EX-Q1B		EX-Q2	
	Original	Modified	Original	Modified	Original	Modified
Matched	137	169	145	168	145	177
Non-matched	37	3	29	6	36	4
Total	174	174	174	174	181	181

Following the session, I re-examined the categories and their descriptions, and introduced some language modifications as discussed with the raters. Appendix E includes the original and modified categories as well as the frequencies indicating the percentage of participants who selected each category.

EX-Q3, EX-Q4, EX-Q5, EX-Q6 and EX-Q7 Analysis

> EX-Q3: In language, we study about sentence structure because it is useful for forming sentences, about adjectives because they are useful for describing things, etc. Can you think of lessons that you found useful in mathematics? If so, what were those lessons about?

> EX-Q4: Language helps us develop communication skills such as reading, listening, and speaking. Did mathematics help you develop any skills? If so, what are those skills?

> EX-Q5: Educators use number operations, averages and percentages to come up with student grades. In your own profession, do you use mathematics? If so, could you describe how you use it?

EX-Q6: Choose a profession, other than your own, where you think that mathematics is used. Could you describe how mathematics is used in this profession?

EX-Q7: Complete the following statement as in the example: To me language is like a key. It opens the door of communication. To me mathematics is like a

______

Several responses reflected a tendency to address questions at a general level. One common example, within the context of skills that mathematics helps us develop, was the mention of a skill such as "math skill" without elaborating on the type of skill. Another example, in the context of useful lessons, was the mention of a branch of mathematics rather than a particular topic – for example Algebra, Geometry, etc.

For each item, I grouped the responses based on recurring themes and came up with categories and subcategories of responses. For example, in EX-Q3 (most useful lesson), one of the categories of responses was Statistics. The subcategories generated were General, Statistics, Graphs, Probabilities, Averages, as well as Z and F scores. The General subcategory was a result of participant responses that did not clearly identify a particular statistical concept. Appendix F provides details of the categories and their corresponding subcategories, arranged in descending order of frequency.

<u>EX-Q 8 Analysis</u>

EX-Q8: Is there any question that you think we should have asked to get to know more about your views of mathematics?

For this item, a general question not related to any one particular research question, I grouped the questions into three types based on related themes. The three types are presented in a Table 3.9 included in the EX-Q Results section.

EX-Q Results

In this section, I will summarize the findings of the exploratory study as they relate to the three research questions.

Research Question One - Categories of Conceptions

What are learner conceptions of mathematics as a discipline?

Table 3.6 displays the percentages of responses left blank (Incomplete), as well as responses not belonging to any categories due to incompleteness or non-relevance (Non-categorized). It also displays the percentage frequencies per category arranged in descending order. For EX-Q1A and EX-Q1B, the frequency data is based on the final categorization done after the discussion session.

Table 3.6

Categories derived from responses to items of Research Question One

Categories	Incomplete Responses	Non-categorized Responses	Categories of Conceptions	Frequency
EX-Q1A (essence of mathematics)	5%	3%	Numbers and Rules	31%
			Intellectual Exercise	21%
			Life Line	18%
			School Subject	14%
			Symbolic Language	10%
			Language of Science	5%
			Self Expression	1%
EX-Q1B (mathematics as a tool)	5%	7%	Calculation tool	34%
			Practical tool	24%
			Intellectual tool	21%
			Language tool	12%
			Problem Solving tool	7%
			Developmental tool	3%
EX-Q7 (metaphors)	13%	1%	Symbols/Numbers	19%
			Key	17%
			Thing or Object	14%
			Subject	13%
			Means	9%
			Tool	8%
			Entrance	7%
			Activity	7%
			Language	5%
			Calculations and Solutions	4%

Note. Frequencies are rounded to the nearest whole number and arranged in descending order. The percentages in the column titled Frequency are calculated out of the total number of complete responses.

Research Question Two – Categories of Conceptions

What are student conceptions of the utility of mathematics in everyday life?

Table 3.7 displays the percentages of responses left blank (Incomplete), as well as responses not belonging to any categories due to incompleteness or non-relevance (Non-categorized). It also displays the percentage frequencies per category, arranged in descending order. For EX-Q2 (uses of mathematics), the data is based on the final categorization done after the discussion session. Only the matched responses were counted.

Table 3.7

Categories derived from responses to items of Research Question Two

Question	Incomplete Responses	Non-categorized Responses	Categories of Conceptions	Frequency
EX-Q2 (uses of mathematics)	2%	7%	Everyday Life Calculations and Estimations Intellectual Development Work Related Situations Quantities and Measures Academic and Professional Development Statistics Scientific and Technological Innovations Communication	25% 25% 15% 14% 9% 5% 3% 3% 3%
EX-Q3 (most useful lessons)	10%	9%	Arithmetic Algebra Geometry Statistics General Mathematics Calculus Mathematics Foundations Business Mathematics	44% 23% 12% 11% 5% 2% 2% 1%
EX-Q4 (most useful skills)	8%	16%	Intellectual Skills Mathematical Skills Academic, Professional and Life Skills Communication Skills	48% 30% 17% 5%

Note. Frequencies are rounded to the nearest whole number and arranged in descending order. The percentages in the column titled Frequency are calculated out of the total number of complete responses.

Research Question Three – Categories of Conceptions

What are student conceptions of the links between mathematical knowledge and its applications in the work environment?

Table 3.8 displays the percentages of responses left blank (Incomplete), as well as responses not belonging to any categories due to incompleteness or non-relevance (Non-categorized). It also displays the percentage frequencies per category.

Table 3.8

Categories derived from responses to items of Research Question Three

Question	Incomplete Responses	Non-categorized Responses	Categories of Conceptions	Frequency
EX-Q5	16%	30%	Academic Sector	28%
			Everyday Use	25%
			Healthcare Sector	20%
			Business and Finance	17%
			Professional Sector	10%
EX-Q6	21%	16%	Business and Trade	46%
			Engineering	20%
			Money and Banking	17%
			Health Sciences	7%
			Academic	5%
			Science and Humanities	3%
			Other Professions	3%

Note. Frequencies are rounded to the nearest whole number and arranged in descending order. The percentages in the column titled Frequency are calculated out of the total number of complete responses.

Responses ranged from the very general to the specific explanation of applications of mathematics in performing tasks related to the different professions. However, the general responses tended to be more dominant. It was easier to acknowledge the uses than to explain them and link them to a particular mathematics topic.

Keeping in mind that most participants are full time students and do not have work related experiences, several chose the academic sector as their current profession. The choice of everyday use is also a reflection of their student status. The other choices reflect their future careers as well their work placement experience. When asked to think of a profession other than their own, the ones that came to mind were business, trade, and financial services as well as engineering. It may be so because their link with mathematics is more apparent that other professions due to the extensive use of numbers especially in the business and financial sectors.

General Question (Item EX-Q8)

Is there any question that you think we should have asked to get to know more about your views of mathematics?

Table 3.9 displays the question types and the percentages of incomplete or non-categorized responses.

Table 3.9

Types of questions for EX-Q8

Question	Incomplete Responses	Non-categorized Responses	Types of Questions
EX-Q8	23%	1%	1. Learning and Teaching Mathematics Why do students find mathematics difficult? How can teachers improve their teaching methods so mathematics can be made easy? 2. Liking and Enjoying Mathematics Do students enjoy mathematics, and how can we help them enjoy it? Do students like mathematics? 3. Value and Use of Mathematics? How much mathematics does a person like to learn? Why do we study mathematics? Does it help us in everyday life? Why does mathematics exist?

While the questions related to the value and use of mathematics were addressed in the questionnaire, the first and second types reflected learner concerns and attitudes rather than conceptions. The following examples may serve to illustrate this point:

'Why does mathematics exist?'
'When will they drop the subject of mathematics?'
'Is there a simpler way to teach Algebra?'
'Do you like mathematics or not?'

The Pilot Stage

IN-Q Development

Using the categories derived from the responses to EX-Q items, I developed the initial questionnaire IN-Q. IN-Q included eight multiple choice items, one item combining open – ended and multiple choices, and one ranking item. In addition, there were two open – ended items. I also included five open – ended questions for formative evaluation purposes:

F1. Was the time given to answer this questionnaire adequate?

F2. Was the font clear?

F3. Was there enough space to write your response?

F4. Would you have liked to have space to write your own response if it did not correspond to any of the options?

F5. Did you have any difficulty reading or understanding the instructions on this questionnaire? Can you please explain what those difficulties were?

For some items, I included additional categories from the literature, or deleted categories with very low frequencies. To illustrate the procedure for the development of the questionnaire items, I will use examples from items corresponding to EX-Q1 (essence of mathematics), EX-Q3 (most useful subjects) and EX-Q7 (metaphors).

Corresponding to EX-Q1, I used the EX-Q1A categories (essence of mathematics) with the highest frequencies: Numbers & Rules (31%), Intellectual Exercise (21%), Life Line (19%), School Subject (14%), Symbolic Language (10%) and Language of Science (5%). I combined the Lifeline and School Subject categories in one item because of their emphasis on applying math in a variety of contexts. I added a category which appears in the literature, mathematics as a 'Form of Art'. Some references to mathematics as an art are found in *Essays in Humanistic Mathematics*, edited by Alvin White. Ness (1993) quotes a description of mathematics found in the *Lawrence University Catalogue*: "Mathematics is the natural home of both abstract thought and laws of nature. It is at once pure logic and creative art" (p. 51). Wales (1993) quotes the famous mathematician G. Hardy as saying: "Real mathematics... must be justified as an art if it is to be justified at all" (p. 33). He also quotes G. Spencer-Brown as saying: "That mathematics, in common with other art forms, can lead us beyond ordinary existence, is no new idea." (p. 32)

The resulting multiple choice item IN-Q2 is displayed in Table 3.10. Next to each multiple choice statement of IN-Q2, I have included the corresponding categories of EX-Q1A.

Table 3.10

<u>IN-Q2 derived from EX-Q1A categories</u>

IN-Q2	Categories of EX-Q1A
Select the statement that best describes your views of mathematics. Select only one statement. Mathematics is	
a group of numbers and rules for doing calculations and arithmetic problems	Numbers and Rules
a mental activity that helps develop several intellectual abilities	Intellectual Exercise
a subject used in the study of other subjects, in everyday life and in the workplace	Lifeline/School Subject
a symbolic language that expresses relations between shapes and measures	Symbolic Language
the language of science used to describe the physical world	Language of Science
a form of art that enhances our creativity and imagination	Form of Art

Corresponding to EX-Q7, I developed a combined open ended and multiple choice item where the participants were instructed to select a category of metaphor and complete the blank statement accordingly. I included the metaphor categories with the highest frequencies or the most meaningful: key to..., collection of things or objects, means for ...., tool for..., language for .... I added a category for other choices. The resulting item IN-Q3 is displayed in Table 3.11.

Table 3.11

<u>IN-Q3 derived from EX-Q7 categories</u>

IN-Q3
Select the statement that best describes what mathematics is to you. Complete this statement with a suitable word or phrase. To me mathematics is ☐ a key to ... ☐ a collection of ... ☐ a means for ... ☐ a tool for ... ☐ a language for ... ☐ other ...

For EX-Q3, I developed one item for the main categories – the mathematics subjects, and one for each related topic. In developing the item on the main categories, I used the following categories: Arithmetic (44%), Algebra (23%), Geometry (12%), Statistics (11%) and Calculus (2%). I did not use the others because they were encompassed by the ones already included. I added a category for Trigonometry as it is an integral part of school mathematics. The resulting multiple choice item IN-Q6 is displayed in Table 3.12.

Table 3.12

<u>IN-Q6 derived from EX-Q3 categories</u>

IN-Q6
Arrange the mathematical subjects in order of importance. The subject that you think is the most useful should be ranked as (1). ☐ Arithmetic ☐ Algebra ☐ Calculus ☐ Statistics ☐ Geometry ☐ Trigonometry

For one of the subcategories of Arithmetic, I developed an item using the categories with the highest frequencies: Basic Operations, Ratio and Proportion, Number and Place Value, Fractions, as well as Percentages and Applications. I added the topic of Ordering and Comparing since it is one of the basic numerical skills. The resulting multiple choice item IN-Q7 is displayed in Table 3.13.

Table 3.13

IN-Q7 derived from EX-Q3 categories

IN-Q7
Arrange the following lessons in order of importance. The lesson that you think is most useful should be ranked as (1). In Arithmetic ☐ The Four Basic Operations ☐ Ratio and Proportion ☐ Reading and Writing Numbers ☐ Fractions ☐ Percentages and Applications ☐ Ordering and Comparing Numbers

IN-Q Administration and Analysis

I conducted the pilot study with a total of 16 participants enrolled in a diploma program at one of the colleges. The sample consisted of 3 males and 13 females between the ages of 18 and 30. The pilot study was conducted with two small groups of 7 and 9 participants, on two consecutive days. Prior to administration, I explained the purpose of pilot study and gave the participants the following instructions:

1. This questionnaire is not for evaluation purposes.
2. It is strictly confidential. You do not need to write your name.
3. It asks for your own views. Please feel free to express them.
4. The duration is 20-25 minutes.

The administration was followed by a brief group discussion regarding the questionnaire in terms of clarity, time, and design.

IN-Q Results

The participants found the questionnaire clear and easy to follow, the time was adequate and the display was satisfactory. Some had problems with the meaning of some mathematical terms. Some found IN-Q3 (metaphors), IN-Q6 (most useful lessons) and IN-Q7 (most useful topics) difficult to follow and respond to. One participant had an inquiry regarding one of the items in the instructions section.

I introduced modifications to the writing style. One such modification was adding the statement: 'Select only one statement' to the instructions, when relevant, to make sure that participants select only one statement. For example, the statement: 'Select the statement that best describes your view of how mathematics developed', was changed to 'Select the statement that best describes your view of how mathematics developed. Select only one statement'.

I also modified the order of the questionnaire sections. One basic modification was moving the instructions section to the end of the questionnaire. Towards the end of a questionnaire, participants feel tired and the instructions section does not require a lot of concentration.

Another type of modification involved the items themselves. I replaced some items and changed the format of others. Some basic changes include items IN-Q3, IN-Q6 and IN-Q7. I will use these items for illustration.

Item IN-Q3 corresponding to EX-Q7 (metaphors) was replaced with an open – ended statement similar to the original item found in EX-Q7. IN-Q3 was confusing for the participants in that some thought that they had to answer every one of the metaphors. Also, some found that it limited their ability to express themselves. An open ended item provides richer data and more flexibility for participants. Table 3.14 displays the original item, IN-Q3 and the modified item.

Table 3.14

Modifications to IN-Q3

IN-Q3	IN-Q3 Modified
Select the statement that best describe what mathematics is to you. Complete this statement with a suitable word or a phrase. To me mathematics is ☐ a key to ... ☐ a collection of ... ☐ a means for ... ☐ a tool for ... ☐ a language for ... ☐ Other	Complete this statement with a phrase that expresses how you think of mathematics. To me mathematics is like a (n) _____

Item IN-Q6 corresponding to EX-Q3 (most useful lessons) was replaced by a multiple choice item since the participants found the ranking style item hard to handle. For the main categories of subjects, Calculus and Trigonometry were excluded because they had a very low frequency. Table 3.15 displays the original item, IN-Q6 and the modified item.

Table 3.15

Modifications to IN-Q6

IN-Q6	IN-Q6 Modified
Arrange the mathematical subjects in order of importance. The subject that you think is most useful should be ranked as (1) ☐ Arithmetic ☐ Calculus ☐ Geometry ☐ Algebra ☐ Statistics ☐ Trigonometry	Select the mathematical subject that you use most. Select only one subject. ☐ Arithmetic ☐ Geometry ☐ Algebra ☐ Statistics

For the topics for each category, I developed a combined checklist for all four subjects instead of one item, in an attempt to simplify and shorten the questionnaire. Table 3.16 includes the modification to item IN-Q7.

Table 3.16

Modifications to IN-Q7

IN-Q7 modified	
Select the mathematical topic that you think is most useful. You may select more than one topic.	
☐ Using Charts ☐ Solving Equations ☐ Calculating Amounts ☐ Estimating Measurement ☐ Reading and Writing Numbers ☐ Recognizing Basic Shapes ☐ Working with Fractions ☐ Working with Ratios ☐ Calculating Area, Perimeter, etc.	☐ Working with Averages ☐ Working with Percentages ☐ Working with Square Roots ☐ Comparing and Ordering Numbers ☐ Knowing Theorems and Proofs ☐ Using Formulas to Calculate Quantities ☐ Solving Word Problems ☐ Using Probabilities

The Final Stage

FN-Q Development

Based on the results of the previous stages, the development of the final questionnaire underwent several refinements. Furthermore, in an effort to establish the internal consistency of the multiple choice questionnaire items, I developed a matching form for each item, with corresponding statements. I used the Chi-Squared test to check the internal consistency of each item by comparing individual responses to each pair of item and matching item. For the item to have internal consistency, the number of matching responses for each pair should be significantly higher than the number of non-matching responses.

I will illustrate the development of the matching forms with their corresponding statements by using an example from item FN-Q12. I kept a similar sentence structure and layout, but replaced every statement representing a category with an example. For example 'dealing with day to day routines' was replaced by 'telling the time', 'advancing science and technology' was replaced by 'finding new ways to improve our world', etc. For each statement, I tried to develop a matching statement that represented the same category, through using a relevant example. Table 3.17 shows the original item FN-Q12 with the ten statements representing its categories of responses, and the matching form FN-Q1 with its ten matching statements. The matching statements are displayed next to the original ones.

Table 3.17

FN-Q12 and its matching form FN-Q1

Item FN-Q12	Matching Form FN-Q1
Select the statement that best describes your view on the usefulness of mathematics. Select only one statement. I find mathematics useful for	Select the statement that best describes how mathematics helps you. Select only one statement. I find mathematics helpful in
☐ dealing with day to day routine	☐ telling the time
☐ dealing with calculations and estimations	☐ finding out how much money is left after shopping
☐ developing intellectual skills	☐ dealing with difficult situations that require analysis
☐ dealing with a variety of work related tasks	☐ reading task and duties schedule
☐ doing activities such as puzzles and games	☐ solving puzzles in magazines or newspapers
☐ understanding government policies	☐ understanding how the social security system works
☐ doing art activities such as drawing and design	☐ designing patterns for weaving
☐ academic and professional development	☐ taking courses to improve some computer skills
☐ advancing science and technology	☐ finding new ways to improve our world
☐ communicating ideas related to numbers and measurement	☐ describing distances between places

The final questionnaire FN-Q included 14 items: ten multiple choice which consisted of five pairs of items and their matching forms, two checklist items, and two open ended questions. I also left a space for remarks, if any. Appendix D demonstrates the correspondence of the FN-Q items to EX-Q and IN-Q items as well as to the three research questions.

Prior to the administration of the questionnaire, I conducted a formative evaluation to check the language and writing style, as well as the correspondence of the item and its matching form. The participants were two faculty members and a program supervisor. One of the participants is a mathematics teacher; the other two are English Language teachers. The modifications are based on the result of the formative evaluation. They are of three types, other than language modifications: writing style, type of example used, and ordering the questionnaire items.

Modifications to FN-Q

For the pairs FN-Q7 and FN-Q10 (essence of mathematics) as well as FN-Q6 and FN-Q13 (evolution of mathematics), the raters were able to select the matching statements with a 100% accuracy. The raters suggested two modifications to the examples provided in FN-Q10.

1. 'Explaining things such as rules of gravity' was introduced, instead of 'expressing scientific ideas' to represent the use of mathematics as a language of science. This is a more specific example. It is also common knowledge.

2. "Developing our ability to think' was introduced, instead of 'developing skills such as critical thinking' as an example of how mathematics as mental exercise develops our intellectual ability. This modification is a better representation of intellectual ability as it is more comprehensive.

For the pair FN-Q12 and FN-Q1 (uses of mathematics), the raters were able to select the matching items with 40% accuracy. Four of the ten pairs of corresponding statements were matched correctly by all three raters, three pairs of the statements were matched correctly by two raters, and the remaining three differed amongst the three raters. This suggested the need for changing some examples.

1. "Recording telephone numbers or reading the time' was replaced by one specific example 'telling the time', to represent dealing with day to day routines.

2. 'Comparing areas of different cities' was replaced by 'describing distances between places' to represent communicating ideas related to numbers and measurement. The latter is a more practical example used quite often in our everyday life.

For the pair FN-Q9 and FN-Q2 (most useful skills), the raters were able to select the matching statements with 50% accuracy. Two of the four pairs of corresponding statements were matched correctly by all three raters, and the remaining two were matched correctly by two raters. For the pair FN-Q3 and FN-Q8 (most useful subjects), the raters were able to select the matching statements with 50% accuracy. Two of the four pairs of corresponding statements were matched correctly by all three raters, and the remaining two were matched correctly by two raters. In both cases it was the same rater whose ratings differed from the other two. The rater expressed having some difficulty with the terminology of items such as variables. 'Solving equations with variables' was replaced by 'solving equations'.

The raters also suggested starting the questionnaire with a concrete item rather than an abstract one. In particular, they were referring to item relating to the evolution of mathematics, which was the first item in IN-Q. This final modification was introduced.

CHAPTER IV

FINAL QUESTIONNAIRE ADMINISTRATION & RESULTS

FN-Q Administration

After incorporating the suggested modifications, the final questionnaire was ready. I administered FN-Q on two different campuses. On one campus, there were two groups of participants enrolled in a customer based one year program. One group had just started the program while the second group were in their last semester. On the second campus, there were seven sections of the same group, enrolled in their first of a four year college diploma program. In all, the sample included 238 participants, all female students. The age range was 16 to 41 years old. I divided the sample into groups using age, academic level and work experience, for post analysis purposes.

The sample was divided into three groups according to Age. Table 4.1 shows the distribution by Age Group.

Table 4.1

Distribution by Age Group

Age Group	Description	Frequency
Group 1: 16-19	16-19 years old	81
Group 2: 20-29	20-29 years old	115
Group 3: 30+	30 years or older	28
Incomplete/Missing data		14
Total Number of Participants		238

The sample was divided into three groups according to Academic Level. Table 4.2 shows the distribution by Academic Level.

Table 4.2

Distribution by Academic Level

Academic Level	Description	Frequency
Group 1: < HS	Less than High School	40
Group 2: HS	High School	175
Group 3: >HS	Post High School	14
Incomplete/Missing Data		9
Total Number of Participants		238

The sample was divided into two groups according to Work Experience. Table 4.3 shows the distribution by Work Experience.

Table 4.3

Distribution by Work Experience

Work Experience	Description	Frequency
Group 1: No Exp.	No work experience	191
Group 2: Exp.	Work experience	47
Total Number of Participants		238

On the first campus, the questionnaire administration was done in one week. On the second campus, it was done in about two weeks. The questionnaires were distributed and collected by the mathematics teachers during classes. Students were given 25 minutes to respond, extended if requested. The teachers were instructed to only answer meanings of terms that participants had difficulty with. Also, students were requested not to discuss their responses with each other.

FN-Q Results

I used MS-EXCEL to record the data from FN-Q, and perform the data analysis per item for the whole sample. I also checked the internal consistency of the five matching pairs of multiple choice items using the Chi-Squared test. The results indicate a significant difference between the matching and the non-matching responses in all cases. For the pairs FN-Q1 and FN-Q12 (uses of mathematics), FN-Q2 and FN-Q9 (most useful skills), as well as FN-Q7 and FN-Q10 (essence of mathematics) the difference was in favor of the non-matched responses. For the pairs FN-Q3 and FN-Q8, (most useful subjects), and FN-Q6 and FN-Q13 (evolution of mathematics), the difference was in favor of the matched responses (see Tables 4.4.1 to 4.4.5 for details). For purposes of testing, the ratings per pair were considered matching when the learner selected the matching statements for the pair of items.

Table 4.4.1

Chi-Squared test for FN-Q1 & FN-Q12 (uses of mathematics)

	Matched Responses	Non-matched Responses
Observed Frequency	46	173
Expected Frequency	109.5	109.5
Chi-Squared Value *	73.6484	

* Critical value as calculated by EXCEL function CHIINV = 6.6349 for one tailed probability p= 0.01 with df = 1

Table 4.4.2

Chi-Squared test for FN-Q2 & FN-Q9 (most useful skills)

	Matched Responses	Non-matched Responses
Observed Frequency	88	142
Expected Frequency	115	115
Chi-Squared Value *	12.6783	

* Critical value as calculated by EXCEL function CHIINV = 6.6349 for one tailed probability p= 0.01 with df = 1

Table 4.4.3

Chi-Squared test for FN-Q3 & FN-Q8 (most useful subjects)

	Matched Responses	Non-matched Responses
Observed Frequency	184	51
Expected Frequency	117.5	117.5
Chi-Squared Value *	74.2723	

* Critical value as calculated by EXCEL function CHIINV = 6.6349 for one tailed probability p= 0.01 with df = 1

Table 4.4.4

Chi-Squared test for FN-Q6 & FN-Q13 (evolution of mathematics)

	Matched Responses	Non-matched Responses
Observed Frequency	172	58
Expected Frequency	115	115
Chi-Squared Value *	56.5403	

* Critical value as calculated by EXCEL function CHIINV = 6.6349 for one tailed probability p= 0.01 with df = 1

Table 4.4.5

Chi-Squared test for FN-Q7 & FN-Q10 (essence of mathematics)

	Matched Responses	Non-matched Responses
Observed Frequency	92	137
Expected Frequency	114.5	114.5
Chi-Squared Value *	8.8428	

* Critical value as calculated by EXCEL function CHIINV = 6.6349 for one tailed probability p= 0.01 with df = 1

Next, I grouped the data using age, academic level and work experience as discussed in the previous section: FN-Q Administration. I performed a post analysis of the data to explore the possible relevance of the variables to the responses. I will present the results of the post analysis following the FN-Q Results section. Refer to Appendix G for detailed results for each pair of item and matching item.)

For each of the three research question, I will present the results of the corresponding FN-Q items (refer to Appendix D to view the correspondence between the three research questions and the FN-Q items).

The frequencies in Tables 4.5 to 4.13 are rounded to the nearest whole number. A few are rounded to one decimal place if the result of the first rounding is 0 (e.g. if the original value is 0.3, the result of rounding to a whole number is 0). They are all arranged in descending order.

Items related to Research Question One

What are learner conceptions of mathematics as a discipline?

The items corresponding to this question are the pairs FN-Q7 and FN-Q10 (essence of mathematics), FN-Q6 and FN-Q13 (evolution of mathematics) as well as item FN-Q4 (metaphors).

Table 4.5 presents the frequencies arranged in descending order of the original item FN-Q7 on the essence of mathematics.

Table 4.5

Responses to FN-Q7 & FN-Q10 (essence of mathematics)

Responses	Frequency	
	Item	Matching Item
Mathematics is a subject used in everyday life and at work	37%	38%
Mathematics is a mental exercise that helps develop intellectual abilities	18%	32%
Mathematics is a mental activity used to develop problem solving techniques	13%	5%
Mathematics is a group of numbers and rules for doing calculations	10%	11%
Mathematics is a school subject used in learning other subjects and for future studies	9%	3%
Mathematics is a symbolic language that expresses relations between objects and quantities	5%	2%
Mathematics is an activity that allows us to express our creativity	4%	5%
Mathematics is a language of science used to describe the physical world	1%	1%

Table 4.6 presents the frequencies arranged in descending order of the original item FN-Q6 on the evolution of mathematics.

Table 4.6

Responses to FN-Q6 & FN-Q13 (evolution of mathematics)

Responses	Frequency	
	Item	Matching Item
People discovered mathematics as they attempted to understand the world around them	64%	49%
People created mathematics to address their practical needs	35%	48%

Table 4.7 presents the frequencies arranged in descending order of item FN-Q4 on metaphors.

Table 4.7

Responses to FN-Q4 (metaphors)

Metaphor	Item Frequency
Subject	39%
Skills	12%
Thing	7%
Life Necessity	6%
Number or Symbol	5%
Way or Method	3%
Operations	3%
Group or Collection	3%
Equations	3%
Procedure or Rule	2%
Mental Activity or Exercise	2%
World or Sea	2%
Domain of thinking	1%
Challenge	1%
Problems or Solutions	1%
Headache or Plague	1%
Language	1%
Statistics	0.4%
Conversions	0.4%
Science	0.4%
Incomplete	8%

Summary of Results – Research Question One

In this section, I will present a summary of the findings related to Research Question One based on the results recorded in Tables 4.5, 4.6 and 4.7.

Essence of Mathematics

The conception of mathematics as a school subject is held by 46% of the students and is distributed as follows: 37% are of the view that it is used in everyday life and at work, while 9% think that it is used for learning other subjects and for future studies.

The conception of mathematics as a mental exercise or activity is held by 31% of the students and is distributed as follows: 18% are of the view that mathematics helps develop intellectual abilities, while 13% think that mathematics is used to develop problem solving techniques.

The conception of mathematics as a group of numbers and rules for doing calculations is held by 10% of the students.

The conception of mathematics as a language was held by 6% of the students and is distributed as follows: 5% view mathematics as a symbolic language for expressing relations between objects and quantities, while 1% think of mathematics as a language of science used to describe the physical world.

The conception of mathematics as an activity that allows us to express our creativity was held by 4% of the students.

Evolution of Mathematics

The conception that mathematics was discovered by people as they attempted to understand the world around them was held by 64% of the students, while 35% think that mathematics was created by people to address their practical needs.

Metaphors of Mathematics

The metaphor holding the highest frequency is the 'subject' metaphor (39%), followed by the 'skills' metaphor (12%), the 'thing' metaphor (7%), the 'life necessity' metaphor (7%) and the 'numbers or symbols' metaphor (5%). All other metaphors account for less than 5% each. In the following paragraphs, I will present extracts of the statements used by some participants to complete the item. They will serve to illustrate the meaning of each of these metaphors.

For the 'subject' metaphor, which held both the highest ranking and the largest percentage relative to all other metaphors, several participants used adjectives to describe different aspects of mathematics. For example, some stressed the importance of mathematics and its role:

An essential subject to learn calculations
An important subject used in everyday life
A useful school subject
An essential subject for all domains

Others reflected their attitudes towards mathematics as a subject, both favorable and non-favorable:

Interesting and enjoyable subject ...
A difficult subject that cannot be understood
Is my favorite subject
An enjoyable and entertaining subject

For the 'skills' metaphor, the statements used by the participants reflected skills such as mathematical, intellectual, practical, and communication skills.

Examples of mathematical skills included:

Mathematics helps us know numbers
Understand equations ...,
Helps us calculate...

Examples of practical skills included:

Knowing numbers and understanding bank transactions...
Reading and writing numbers and dealing with others:..
Solving problems related to buying ...

Examples of intellectual skills included:

To deal with difficult situations ...
A useful way to make the mind work
To activate the mind ...

Examples of communication skills included:

Is communicating with others
Communications...
Knowing how to communicate...

For the 'thing' metaphor, the statements included the use of the term 'thing' to refer to mathematics without determining its nature i.e. to refer to an undefined object. Along with this, participants used adjectives with both positive and negative connotation such as:

A very important thing for everyday and work
An enjoyable thing, entertaining...
A very annoying and boring thing
A realistic thing in our lives and terrifying when it cannot be dealt with properly ...

The statements also include word such as 'something' or 'everything':

Something that I would like to learn and use in my everyday life,
Everything in this world ...

For the 'life necessity' metaphor, few statements specifically mention the term 'life necessity', but several implied it, such as:

Part of everyday life that helps us know everything we need regarding time, money ...
Important for everyday and professional life
We need it for all our job related work, in the street, market, scientific games,
Basic for professional life...
We need mathematics everywhere

For the 'number or symbol' metaphor, participants sometimes included some uses in their statements such as:

Numbers used to solve equations
Numbers that I use in my everyday life ...

Some participants included numbers or symbols with a list of other mathematical objects as in:

Numbers, equations and ...
Numbers, operations and rules...
Equations and symbols to be solved

Items related to Research Question Two

What are student conceptions of the utility of mathematics in everyday life?

The items corresponding to this question are the pairs FN-Q1 and FN-Q12 (uses of mathematics), FN-Q2 and FN-Q9 (most useful subjects), FN-Q3 and FN-Q8 (most useful skills) as well as items FN-Q5 (checklist of topics) and FN-Q11 (checklist for types of skills).

Table 4.8 presents the frequencies arranged in descending order of the original item FN-Q12 on the uses of mathematics.

Table 4.8

Responses to FN-Q12 & FN-Q1 (uses of mathematics)

Responses	Frequency	
	Item	Matching Item
Mathematics is useful for dealing with day to day routines	28%	8%
Mathematics is useful for dealing with calculations and estimations	19%	17%
Mathematics is useful for developing intellectual skills	14%	34%
Mathematics is useful for communicating ideas related to numbers and measurements	12%	3%
Mathematics is useful for advancing science and technology	8%	24%
Mathematics is useful for doing activities such as puzzles or games	5%	3%
Mathematics is useful for dealing with a variety of work related tasks	5%	2%
Mathematics is useful for doing art activities such as drawing and design	3%	0%
Mathematics is useful for academic and professional development	2%	4%
Mathematics is useful for understanding government policies	0%	1%

Table 4.9 presents the frequencies arranged in descending order of the original item FN-Q9 on the most useful skills.

Table 4.9

Responses to FN-Q9 & FN-Q2 (most useful skills)

Responses	Item Frequency	Matching Item Frequency
Intellectual Skills	58%	50%
Practical Skills	23%	7%
Mathematical Skills	10%	25%
Communication Skills	8%	16%

The most useful types of skills that mathematics helps develop are listed in descending order of frequency in Table 4.10. It is important to note that participants were allowed to select as many statements as they thought relevant. Thus the frequencies listed indicate the percentage of participants who selected that particular statement out of the total number of participant selections.

Table 4.10

Responses to FN-Q11 (most useful types of skills)

Responses	Item Frequency
thinking skills	61%
attention and concentration	59%
doing calculations	56%
dealing with others in markets	53%
everyday life skills	52%
calculator and computer skills	45%
solving problems	39%
memory skills	35%
solving equations	34%
learning skills	34%
communicating through numbers	26%
scientific skills	24%
reading and interpreting charts	23%
measuring	23%
business skills	19%
academic skills for learning other subjects	13%
communicating with nature	12%
language skills	9%

Table 4.11 presents the frequencies arranged in descending order of the original item FN-Q3 on the most useful subjects.

Table 4.11

Responses to FN-Q3 & FN-Q8 (most useful subjects)

Responses	Frequency	
	Item	Matching Item
Arithmetic is the most useful subject	82%	88%
Algebra is the most useful subject	8%	8%
Statistics is the most useful subject	7%	3%
Geometry is the most useful subject	3%	0%

The most useful topics are listed in descending order of frequency in Table 4.12. It is important to note that participants were allowed to select as many statements as they thought relevant. Thus the frequencies listed indicate the percentage of participants who selected that particular statement out of the total number of participant selections.

Table 4.12

Responses to FN-Q5 (most useful topics)

Responses	Item Frequency
Calculating Amounts	60%
Solving Word Problems	53%
Reading and Writing Numbers	46%
Solving Equations	43%
Working with Percentages	41%
Comparing and Ordering Numbers	26%
Working with Fractions	24%
Working with Ratios	22%
Using Charts	21%
Calculating Area, Perimeter, etc	19%
Knowing Theorems and Proofs	18%
Estimating Measurements	16%
Working with Averages	14%
Working with Square Roots	14%
Using Formulas to Calculate Quantities	13%
Using Probabilities	8%
Recognizing Basic Shapes	7%

Summary of Results – Research Question Two

In this section, I will present a summary of the findings related to Research Question Two based on the results recorded in Tables 4.8 to 4.12.

Uses of Mathematics

The conceptions of mathematics as useful for dealing with day to day routines, dealing with a variety of work related tasks, and for academic and professional development are held by 28%, 5% and 2% of students respectively.

The conception of mathematics as useful for dealing with calculations and estimations is held by 19% of the students.

The conception of mathematics as useful for developing with intellectual skills is held by 14% of the students.

The conception of mathematics as useful for communicating ideas related to numbers and measurements is held by 12% of the students.

The conception of mathematics as useful for advancing science and technology is held by 8% of the students.

The conceptions of mathematics as useful for doing activities such as puzzles or games, or arts activities such as drawing and design are held by 5% and 3% of the students respectively.

Most Useful Skills Developed through Mathematics

Intellectual skills held the highest frequency (58%), followed by Practical (23%), Mathematical (10%) and Communication skills (8%).

Intellectual skills are skills related to thinking, learning, understanding, etc. Thinking skills (61%), and attention and concentration skills (59%) held the highest two frequencies amongst the checklist of different types of skills. Other intellectual skills included were problem solving skills (39%), memory skills (35%) and learning skills (34%).

Practical skills are skills needed in everyday life, at work or in studies. The practical skills included everyday life skills (52%), calculator and computer skills (45%), scientific skills (24%), business skills (19%) and academic skills for learning other subjects (13%).

Mathematical skills are skills in performing certain mathematical procedures. These included doing calculations (56%), solving equations (34%), reading and interpreting charts (23%) and measuring skills (23%).

Communication skills are skills in expressing one's self or ideas. These include dealing with others in the markets (53%), communicating through numbers (26%), communicating with nature (12%) and language skills (9%).

Most Useful Mathematics Lessons and Topics

Arithmetic held the highest frequency (82%) followed by Algebra (8%), Statistics (7%) and least Geometry (3%).

An Arithmetic topic held the highest frequency in the checklist of the most useful mathematics topics – calculating amounts (60%). Other topics included reading and writing numbers (46%), working with percentages (41%), comparing and ordering numbers (26%), working with fractions (24%) and working with ratio (22%).

An Algebra topic held the second highest frequency - solving word problems (53%). Other topics include solving equations (43%), working with square roots (14%) and using formulas to calculate quantities (13%).

Statistics topics included using charts (21%), working with averages (14%) and using probabilities (8%).

Geometry topics included calculating area, perimeter etc. (19%), knowing theorems and proofs (18%), estimating measurement (16%) and recognizing basic shapes (7%).

Items related to Research Question Three

What are student conceptions of the links between mathematical knowledge and its applications in the work environment?

Table 4.13 lists examples of daily and work related uses of mathematics as identified by the participants in descending order of frequency.

Table 4.13

Responses to FN-Q14 (uses of mathematics for daily and work related tasks)

Daily Task Examples	Frequency
Market/Shopping	28%
Time/Calendar	20%
Budget/Expenses	17%
Calculating Amounts	8%
Using Numerical Information	6%
Shapes and Measurement	3%
Studies	2%
Personal Banking	2%
Estimating Amounts	1%
Cooking/Baking	1%
Counting	1%
Mental Ability	1%
Using Percentages	0.7%
Calculator/Computer	0.7%
Problem Solving	0.3%
Missing	8%

Work-related Task Examples	Frequency
Accounting	28%
Using Numerical Information	6%
Using Tabular Information	6%
Calculating Amounts	5%
Time/Calendar	4%
Banking	4%
Statistical Analysis	4%
Problem Solving	3%
Budgets/Tenders	3%
Studies	2%
Mental Ability	2%
File Organization	1%
Academic Evaluation	1%
Computer/Calculator	1%
Counting Objects	1%
Development	1%
Engineering Drawings	1%
Trade/Commerce	1%
Ordering Numbers	1%
Missing	26%

Summary of Results – Research Question Three

In this section, I will present a summary of the findings related to Research Question Three based on the results recorded in Tables 4.13.

The types of examples included general ones such as 'calculating amounts' as well as specific ones such as 'calculating the remaining days in a month'. Few participants mentioned the mathematical topic associated with a particular use such as 'when shopping we use addition to calculate our total spending' or 'we use numbers in organizing our work and our files by number and date'. Several responses mentioned more than one type of example such as 'knowing time schedules, calculating vacation days and salaries'. Also, there was some overlap between categories such as in the case of Time/Calendar and Calculating Amounts. For example, if a participant uses the example of 'calculating time', this was labeled as belonging to the Time/Calendar type since it mentions a specific use. Calculating Amounts was reserved for general calculations such as 'money calculations' which does not particularly specify the context, or such as '... calculate things'.

There were 15 types of examples provided on daily task, and 18 on work related tasks. The percentage of missing responses on work related tasks (26%) was nearly three times as much as on daily tasks (8%). This may be due to the fact that the majority of participants have little or no work experience.

Daily Task Examples

Market/Shopping examples ranked highest (28%) followed by Time/Calendar examples (20%), Budget/Expenses examples (17%), Calculating Amounts (8%) and Using Numerical Information (6%). All other types had less than 5% frequency.

In the following paragraphs, I will provide extracts from participant responses to describe these categories.

Market/Shopping examples include:

Operations for buying and selling in the market
Use addition when buying things from the supermarket
When we go to the market to buy things and when we go home, we add the prices to know how much we spent
Dealing with markets and shops

Time/Calendar examples include:

Knowing the difference in time between countries, dates
Determining time
In using the watch to know the time
We use numbers to know the dates and days
In calculating and months, in knowing TV program times ...

Budget/Expenses examples included the mention of bills, expenses and budget such as:

Paying monthly bills
To calculate or estimate family income
How to budget the salary
Expense bills, home expenses

Calculating Amounts include examples on general calculations such as:

We use addition to calculate things
Calculating amounts

Using Numerical Information included examples on the use of numbers in a variety of contexts such as:

Reading bills
Reading a number on a care plate or a journal
Reading phone bills, post, magazines,
To allocate phone numbers

Work-related Task Examples

Accounting examples ranked highest (28%). There was a gap between the highest frequency and the second highest – Using Numerical Information (6%). Next, there was Using Tabular Information (6%) and Calculating Amounts (5%). All other types had less than 5% frequency.

In the following paragraphs, I will provide extracts from participant responses to describe these categories.

Accounting examples include salary calculations, expenses ...

To calculate salaries, incentives and deductions
Employee accounts, employee salaries, company accounts,
To calculate money and bills related to work
Monthly salaries, work hours, accounts

Using Numerical Information examples include using numbers in a variety of everyday as well as work related contexts:

Numbers in assigning centers for students
Numbering homes and villas
We use numbers for years of export in government or companies
Numbering statements

Using Tabular Information examples include reading and using tables of information:

Knowing time schedules
Organizing work schedules
Tables

Time/Calendar examples include:

Vacation times
Knowing the starting day and time for work
To know time between periods
I use time in determining when to go to my job
To know time and organize it

Post Analysis Results

For purposes of the post analysis, I will be comparing the results of the original multiple choice items across the Age Group, Academic Level and Work Experience (see Tables 4.1, 4.2 and 4.3 for more detail on the division of groups, and Appendix G for the data on frequencies of responses).

Item Responses by Age Groups

Tables 4.14 to 4.18 present the different frequencies of responses to items FN-Q7 (essence of mathematics), FN-Q6 (evolution of mathematics), FN-Q12 (uses of mathematics), FN-Q9 (most useful skills) and FN-Q3 (most useful subjects) per Age Group. The tables are arranged in descending order of frequency for the whole sample.

Table 4.14

Responses to FN-Q7 (essence of mathematics) by Age Group

Responses	Frequency		
	16-19	20-29	30+
Mathematics is a subject used in everyday life and at work	30%	43%	36%
Mathematics is a mental exercise that helps develop intellectual abilities	23%	13%	18%
Mathematics is a mental activity used to develop problem solving techniques	15%	11%	18%
Mathematics is a group of numbers and rules for doing calculations	11%	10%	7%
Mathematics is a school subject used in learning other subjects and for future studies	9%	9%	14%
Mathematics is a symbolic language that expresses relations between objects and quantities	4%	6%	7%
Mathematics is an activity that allows us to express our creativity	6%	4%	0%
Mathematics is a language of science used to describe the physical world	1%	2%	0%

Table 4.15

Responses to FN-Q6 (evolution of mathematics) by Age Group

Response	Frequency		
	16-19	20-29	30+
People discovered mathematics as they attempted to understand the world around them	60%	65%	68%
People created mathematics to address their practical needs	40%	35%	32%

Table 4.16

Responses to FN-Q12 (uses of mathematics) by Age Group

Responses	Frequency		
	16-19	20-29	30+
Mathematics is useful for dealing with day to day routines	23%	33%	29%
Mathematics is useful for dealing with calculations and estimations	17%	21%	18%
Mathematics is useful for developing intellectual skills	17%	11%	18%
Mathematics is useful for communicating ideas related to numbers and measurements	9%	11%	18%
Mathematics is useful for advancing science and technology	10%	9%	4%
Mathematics is useful for doing activities such as puzzles or games	9%	4%	0%
Mathematics is useful for dealing with a variety of work related tasks	7%	4%	0%
Mathematics is useful for doing art activities such as drawing and design	1%	2%	4%
Mathematics is useful for academic and professional development	2%	1%	4%
Mathematics is useful for understanding government policies	0%	0%	0%

Table 4.17

Responses to FN-Q9 (most useful skills) by Age Group

Responses	Frequency		
	16-19	20-29	30+
Intellectual Skills	69%	54%	46%
Practical Skills	14%	25%	29%
Mathematical Skills	7%	10%	18%
Communication Skills	9%	9%	4%

Table 4.18

Responses to FN-Q3 (most useful subjects) by Age Group

Responses	Frequency	Responses	Frequency
	16-19	20-29	30+
Arithmetic is the most useful subject	72%	89%	93%
Algebra is the most useful subject	16%	3%	4%
Statistics is the most useful subject	9%	7%	0%
Geometry is the most useful subject	2%	2%	4%

Item Responses by Academic Level

Tables 4.19 to 4.23 present the different frequencies of responses to items FN-Q7 (essence of mathematics), FN-Q6 (evolution of mathematics), FN-Q12 (uses of mathematics), FN-Q9 (most useful skills) and FN-Q3 (most useful subjects) per Academic Levels. The tables are arranged in descending order of frequency for the whole sample.

Table 4.19

Responses to FN-Q7 (essence of mathematics) by Academic level

Responses	Frequency		
	<HS	HS	>HS
Mathematics is a subject used in everyday life and at work	50%	36%	21%
Mathematics is a mental exercise that helps develop intellectual abilities	13%	18%	36%
Mathematics is a mental activity used to develop problem solving techniques	10%	13%	21%
Mathematics is a group of numbers and rules for doing calculations	10%	10%	7%
Mathematics is a school subject used in learning other subjects and for future studies	8%	10%	0%
Mathematics is a symbolic language that expresses relations between objects and quantities	10%	5%	0%
Mathematics is an activity that allows us to express our creativity	0%	5%	7%
Mathematics is a language of science used to describe the physical world	0%	2%	0%

Table 4.20

Responses to FN-Q6 (evolution of mathematics) by Academic Level

Responses	Frequency		
	<HS	HS	>HS
People discovered mathematics as they attempted to understand the world around them	68%	63%	79%
People created mathematics to address their practical needs	33%	37%	21%

Table 4.21

Responses to FN-Q12 (uses of mathematics) by Academic Level

Responses	Frequency		
	<HS	HS	>HS
Mathematics is useful for dealing with day to day routines	35%	27%	21%
Mathematics is useful for dealing with calculations and estimations	18%	21%	7%
Mathematics is useful for developing intellectual skills	15%	13%	36%
Mathematics is useful for communicating ideas related to numbers and measurements	18%	10%	14%
Mathematics is useful for advancing science and technology	3%	10%	7%
Mathematics is useful for doing activities such as puzzles or games	3%	6%	7%
Mathematics is useful for dealing with a variety of work related tasks	5%	5%	0%
Mathematics is useful for doing art activities such as drawing and design	3%	2%	0%
Mathematics is useful for academic and professional development	3%	2%	0%
Mathematics is useful for understanding government policies	0%	0%	0%

Table 4.22

Responses to FN-Q9 (most useful skills) by Academic Level

Responses	Frequency		
	<HS	HS	>HS
Intellectual Skills	53%	59%	64%
Practical Skills	25%	23%	0%
Mathematical Skills	5%	10%	21%
Communication Skills	15%	7%	7%

Table 4.23

Responses to FN-Q3 (most useful subjects) by Academic Level

Responses	Frequency		
	<HS	HS	>HS
Arithmetic is the most useful subject	80%	82%	93%
Algebra is the most useful subject	0%	9%	7%
Statistics is the most useful subject	13%	6%	0%
Geometry is the most useful subject	5%	2%	0%

Item Responses by Work Experience

Tables 4.24 to 4.28 present the different frequencies of responses to items FN-Q7 (essence of mathematics), FN-Q6 (evolution of mathematics), FN-Q12 (uses of mathematics), FN-Q9 (most useful skills) and FN-Q3 (most useful subjects) per Work Experience Group. The tables are arranged in descending order of frequency for the whole sample.

Table 4.24

Responses to FN-Q7 (essence of mathematics) by Work Experience

Responses	Frequency	
	Exp	No Exp
Mathematics is a subject used in everyday life and at work	38%	36%
Mathematics is a mental exercise that helps develop intellectual abilities	15%	19%
Mathematics is a mental activity used to develop problem solving techniques	19%	12%
Mathematics is a group of numbers and rules for doing calculations	11%	10%
Mathematics is a school subject used in learning other subjects and for future studies	9%	9%
Mathematics is a symbolic language that expresses relations between objects and quantities	6%	5%
Mathematics is an activity that allows us to express our creativity	2%	5%
Mathematics is a language of science used to describe the physical world	0%	2%

Table 4.25

Responses to FN-Q6 (evolution of mathematics) by Work Experience

Responses	Frequency	
	Exp	No Exp
"People discovered mathematics as they attempted to understand the world around them",	70%	62%
"People created mathematics to address their practical needs",	30%	37%

Table 4.26

Responses to FN-Q12 (uses of mathematics) by Work Experience

Responses	Frequency	
	Exp	No Exp
Mathematics is useful for dealing with day to day routines	30%	28%
Mathematics is useful for dealing with calculations and estimations	23%	28%
Mathematics is useful for developing intellectual skills	11%	15%
Mathematics is useful for communicating ideas related to numbers and measurements	11%	12%
Mathematics is useful for advancing science and technology	6%	9%
Mathematics is useful for doing activities such as puzzles or games	6%	5%
Mathematics is useful for dealing with a variety of work related tasks	2%	5%
Mathematics is useful for doing art activities such as drawing and design	4%	2%
Mathematics is useful for academic and professional development	0%	2%
Mathematics is useful for understanding government policies	0%	0%

Table 4.27

Responses to FN-Q9 (most useful skills) by Work Experience

Responses	Frequency	
	Exp	No Exp
Intellectual Skills	55%	58%
Practical Skills	21%	23%
Mathematical Skills	13%	9%
Communication Skills	6%	8%

Table 4.28

Responses to FN-Q3 (most useful subjects) by Work Experience

Responses	Frequency	
	Exp	No Exp
Arithmetic is the most useful subject	91%	79%
Algebra is the most useful subject	2%	9%
Statistics is the most useful subject	4%	8%
Geometry is the most useful subject	0%	4%

Summary of Post Analysis Results

Regarding Age Groups and Work Experience groups, there appears to be little or no differences in the responses to the multiple choice items, especially the ones with the higher frequencies.

Regarding Academic Level, there were differences in the responses for items: FN-Q7 (essence of mathematics), FN-Q12 (uses of mathematics), FN-Q9 (most useful skills) and FN-Q3 (most useful subjects)

Item FN-Q7 (essence of mathematics)

The group with the post high school education viewed mathematics primarily as a mental exercise used to develop intellectual ability while the other two groups viewed it primarily as a subject used in everyday life and work. In addition, no one in this group viewed mathematics as a subject used in learning other subjects, or as a symbolic language that expresses relations between objects and quantities.

Fifty percent of the group with less than high school education viewed mathematics as a subject that is used in everyday life and at work. Thirty six percent and 21% of the other two groups viewed it as so. No one this group viewed mathematics as an activity that allows us to express our creativity.

Item FN-Q6 (uses of mathematics)

The group with post high school education viewed mathematics primarily as useful for developing intellectual skills, while the other two groups viewed it as useful for dealing with day to day routines. In addition, while the other two groups had mathematics as useful for dealing with calculations and estimations as their second frequency, this group had this statement as their fourth. Their second highest frequency was mathematics as useful for dealing with day to day routines. One more thing to note is that no one in this group viewed mathematics as useful for dealing with a variety of work related tasks, doing art activities or for academic and professional development.

Item FN-Q9 (most useful skills)

The group with post high school education did not think that mathematics helps develop practical skills. They attached more importance to mathematical skills relative to the other two groups. The frequency for this statement was 21%, while it was 5% and 10% for the other two groups.

The group with less than high school education attached more importance to communications skills relative to the other two groups. The frequency for this statement was 15%, while it was 7% for the other two groups.

Item FN-Q3 (most useful subjects)

None of the participants in the group with less than high school education viewed Algebra as a useful subject. In addition, this group attached more importance to Statistics as a useful subject, than the other two groups. The frequency for this statement was 13%, while it was 6% for one group and 0% for the other.

Statistics and Geometry were not viewed as useful by the group with post high school education.

CHAPTER V

SUMMARY AND INTERPRETATION

In this chapter, I will present a summary of the findings related to each of the three research questions based on the results discussed in Chapter IV. While the current study may differ from others in the literature in its definition of conceptions, procedures and data collection instruments, its results display common features with other findings. These similarities will be discussed. Next, I will present an overall summary of the major findings and conclude by interpreting the results.

The Research Questions

Research Question One

What are student conceptions of mathematics as a discipline?

Table 5.1 summarizes the findings for Research Question One from three perspectives: essence of mathematics, evolution of mathematics and metaphors of mathematics.

Table 5.1

<u>Summary of Research Question One results (student conceptions of mathematics)</u>

<table>
<tr><th>Essence</th><th>Metaphors</th><th>Evolution</th></tr>
<tr><td>‘school subject’ used in everyday life and at work (37%)</td><td>‘subject’ (39%)</td><td rowspan="4">‘discovered’ as people attempted to understand the world around them (64%)</td></tr>
<tr><td>‘mental exercise’ to develop intellectual ability (18%)</td><td>different types of ‘skills’ (12%)</td></tr>
<tr><td>‘mental activity’ to develop problem solving techniques (13%)</td><td>‘thing’ e.g. important thing, useful thing, (7%)</td></tr>
<tr><td>group of ‘numbers and rules’ for doing calculations (10%)</td><td>‘life necessity’ (6%)</td></tr>
<tr><td>‘school subject’ used in learning other subjects and future studies (9%)</td><td>‘numbers or symbols’ (5%)</td><td rowspan="4">‘created’ to address people’s practical needs (35%)</td></tr>
<tr><td>‘symbolic language’ expressing relations between objects and quantities (5%)</td><td>‘way or method’, ‘operations’, ‘group or collection of objects’ (3% each)</td></tr>
<tr><td>a ‘form of art’ - an activity that allows us to express our creativity (4%)</td><td>‘procedures or rules’, ‘mental activity or exercise’, ‘world or sea’ (2% each)</td></tr>
<tr><td>a ‘language of science’ used to describe the physical world (1%)</td><td>‘domain of thinking’, ‘challenge’, ‘language’, ‘problems or solutions’, ‘headache or plague’, ‘statistics’, ‘conversions’, ‘science’ (1% or less each)</td></tr>
</table>

Notes. The three perspectives essence, evolution and metaphors are derived from responses to the final questionnaire items: FN-Q7 (essence of mathematics), FN-Q6 (evolution of mathematics) and FN-Q4 (metaphors of mathematics).
The percentages represent the response frequencies rounded to the nearest whole number and arranged in descending order per item

Essence of Mathematics – How do students view mathematics?

The majority of students view mathematics as a useful subject or as a form of intellectual activity. Referring to Table 5.1, nearly half the students reported that mathematics is primarily a 'school subject' that is used in everyday life, at work or in pursing current or future studies. About one third viewed mathematics as a 'mental exercise' or 'mental activity' for developing intellectual abilities and problem solving techniques.

Less frequently cited views listed in Table 5.1 include mathematics as a group of 'numbers and rules' for doing calculations, as a 'symbolic language' for expressing relations, a 'language of science' for describing the physical world, or a 'form of art' that allows us to express our creativity.

Frid and White (1995) identified two main conceptions reflecting the views of most students, and consistent with previous research findings. In their study, mathematics was viewed as numbers, rules and formulas. It was also viewed as a logical process or way of thinking. Comparing these conceptions with the ones derived in the current study, three observations come to mind:

- In the current study, the conception most frequently reported emphasizes the essence of mathematics as a 'school subject' but does not specifically discuss its constitution. It discusses the contexts in which mathematics is useful, for example everyday life. Thus it is not possible to align it with one particular conception reported in Frid and White's study.

- The conception of mathematics as a group of 'numbers and rules' for doing calculations shares common features with Frid and White's conception of mathematics as numbers, rules and formulas. A similar view is presented by Countryman (1992) who suggests that students define math narrowly as a collection of rules, facts, numbers, symbols, formulas and right answers.
- The conceptions of mathematics as a 'mental exercise' or 'mental activity' share common features with the second conception in Frid and White's study – mathematics as a logical process or a way of thinking.

Frid and White also identified a third conception, less frequently used – mathematics as a connected hierarchy that studies relationships or patterns. Two less frequently cited conceptions – mathematics as a 'symbolic language' expressing relations between objects and quantities, and as a 'language of science' used to describe the physical world, share some features of Frid and White's third conception, namely, mathematics as a language for expressing different relations.

Oaks presents another perspective on conceptions as they relate to student learning (as cited in Grouws et al. 1996). The results of her study indicate that college students who fail remedial mathematics courses view it as a rote manipulation of symbols. This *dualistic* conception of mathematics is one where mathematics is viewed as a process for finding answers to problems in a specific prescribed way and where problems have a definite answer. They view mathematics as an exact body of knowledge, and view themselves as having little control over it. On the other hand, she defines the *relativist* conception of mathematics where problems may have different answers depending on the situation. While the dualistic conception involves learning mathematics by memorizing, the relativistic conception involves deducing results and processes.

In their own study, Grouws et al. (1996) reported that mathematically talented high school students viewed mathematics as a "system of coherent and inter-related concepts and principles which is continuously growing" (p. 32). Regular students shared the view about mathematics being dynamic and growing; however, they tended to see mathematics more as a "discrete system of facts and procedures that requires more memorizing than thinking" (p. 32).

Oak's dualistic conception of learning as well as the conception held by regular students in Grouws et al. share elements of the commonly held view of mathematics as involving rules, procedures and processes, and thus memorization. This view is mostly reflected in the conception of mathematics as a 'numbers and rules' in the current study.

Essence of Mathematics - Differences in Group Responses

As discussed in Chapter IV, Academic Level appears to be the only one variable revealing some differences in the way the different groups addressed the multiple choice item FN-Q7 (essence of mathematics). However, the three groups varied in size. There were 40 students with less than high school education (<HS), 175 with high school education (HS) and 14 who have post high school education (>HS). To examine the differences in responses, I used the Chi-Squared test. This necessitated combining the (HS) and (>HS) groups due to the restrictions imposed on the values of expected frequencies. As indicated by Popham (1967, p. 296): "The limitation is that no more than 20 per cent of the cells have an expected frequency smaller than 5.0 and no cell has an expected frequency smaller than 1.0. ... If too many small expected frequencies exist, the categories should be combined, unless such combinations are meaningless".

The Chi-Squared test revealed no significant difference between the group responses to the three highest ranking conceptions. Table 5.2 provides the details.

Table 5.2

Chi-Squared test for groups with different academic levels on FN-Q7 (essence of mathematics)

	School Subject	Mental Activity	Numbers and Rules
No High School Observed Frequency	23	9	4
No High School Expected Frequency	21.3	14.5	4.4
High School or more Observed Frequency	83	63	18
High School or more Expected Frequency	84.7	57.5	17.6
Chi-Squared Value *		2.8121	

* Critical value as calculated by EXCEL function CHIINV = 5.9914 for one tailed probability p= 0.05 with df = 2

Notes. School subject, mental activity and numbers and rules are the three highest ranking conceptions of mathematics. They are presented below:
School Subject – mathematics as a school subject used everyday, at work, in future studies, in learning other subjects
Mental Activity – mathematics as a mental activity or exercise for developing intellectual and problem solving abilities
Numbers and Rules – mathematics as a group of numbers and rules for doing calculations

Although there is no evidence of statistically significant differences amongst students with different academic levels in relation to the choice of conceptions, it is interesting to note that mathematics as a 'mental exercise' and 'mental activity' are more highly ranked by the group with post high school education than by the others. On the other hand, mathematics as a 'school subject' used in everyday life, at work, for other subjects and for future studies, is less highly ranked by this group than the other two groups.

Metaphors of Mathematics - How do students describe mathematics?

The majority of students use the 'subject' metaphor listed in Table 5.1 to describe mathematics, e.g. important subject, useful subject etc. This metaphor reflects views on the importance and use of mathematics as well as attitudes towards it. Another metaphor used to describe mathematics is 'skills', listed as the second highly rated metaphor in Table 5.1, where students mention several skills that were classified into four main types: mathematical (e.g. calculations), intellectual (e.g. activating the mind), communication (e.g. communicating with people), and practical (e.g. reading and writing numbers). These skills are very similar to the categories of skills established in Research Question Two (see Table 5.3 for details on categories of skills).

The third metaphor listed in Table 5.1, mathematics as a 'thing', is cited by a fewer number of students. They attach adjectives describing its importance or reflecting their attitudes to it, without actually defining or describing this 'thing'. For example, they refer to mathematics as a useful thing, difficult thing, etc. Two less frequently used metaphors listed in Table 5.1 are: 'life necessity' – where students use different types of statements that imply the need for mathematics, and 'number or symbol' – where they describe mathematics as numbers or symbols along with other objects of mathematics. The remaining metaphors are used by a smaller percentage of the students (3% or less each). These included metaphors such as 'way or method', 'procedures or rules', 'language', etc. (see Table 5.1 for a complete list).

Several metaphors were characterized by using adjectives or nouns reflecting strong emotions towards mathematics. Zaslavsky (1994) does not find the expression of emotion in relation to mathematics surprising: "Many people think of mathematics as one of the most impersonal branches of knowledge, yet it inspires more emotions than any other school subject" (p. 5). Using attitudes or feelings was observed by Gibson (1994) when discussing the use of metaphors to reveal her students' attitudes to mathematics. Gibson distinguishes between two types of learners: *Thinkers* and *Robots*. Thinkers are more active in class and ask a lot of questions. They are not afraid to discuss their ideas. Robots are passive and less likely to engage in debates. She refers to their attitude in the mathematics classroom as "Give me the formula" (p. 7). Her observations about student behaviors in class led her to want to know more about them as learners and about their conceptions of mathematics through asking them to compare mathematics with specific objects. She notes that the metaphors used included both positive and negative components. The negative included a sense of despair and frustration with mathematics while the positive exuded a sense of challenge and enjoyment.

Comparing the attitudes of the Thinkers and the Robots, 60% of Thinkers felt a sense of excitement when doing mathematics, while only 30% of the Robots reported similar emotions. More often, the Thinkers list included words such as 'happy', 'satisfied', and 'challenged', while the Robots list included words such as 'difficult', 'uninteresting'.

Buerk and Szablewski (1993) discuss the use of metaphors as part of journal writing in college mathematics courses. Metaphors provide us with insight into our learners' views. They help us see things as our learners see them. Minuchin talks of metaphors as ways to describe one's experiences and ideas (in Buerk and Szablewski, 1993). They are needed to describe life. Thus, Minuchin remarks, objectivity is not enough for doing so.

Metaphors are subjective by nature. They reflect people's personal views and experiences. They are also subjective in that we can interpret them in different ways – we may read more or less into a metaphor because we are interpreting it through our own experiences. In this study, the categories of metaphors generated are tentative. Several students describe mathematics in general terms as clear from the use of words such as 'thing' – rather than using specific objects. Some tend to use this question to describe their feelings and attitudes regarding mathematics rather than describe mathematics itself. It is important to exercise caution in interpreting metaphors. I will attempt to do in the section summarizing the integrated findings of Research Question One, in light of the other perspectives used.

Metaphors of Mathematics - Differences in Group Responses

In this case, it was not possible to use the Chi-Squared test because a student's response could fall in more than one category. I used the percentage frequencies as the basis of comparison. Comparing groups with different academic levels, the 'subject' metaphor was the most favored by the students with high school education or less. There were differences in ranking the 'skills' metaphor. The students with post high school education ranked this metaphor higher than the other groups. The 'life necessity' metaphor also reflected different emphasis. The group with less than high school education ranked it higher than the other two groups.

Evolution of mathematics – Is mathematics discovered or invented?

The majority of students favored the view that mathematics was 'discovered' by people as they attempted to understand the world around them, rather than 'created' by people to address their practical needs, as reported in Table 5.1. The view of mathematics as 'discovered' shares common features with the Platonic view of mathematics suggested by Ernest (see Chapter 2 – section on Conceptions of Mathematics). The Platonic view portrays mathematics as a discovered static body of knowledge. The view of mathematics as 'created' shares common features with the problem solving view of mathematics. This view portrays mathematics as a dynamic field of human invention which is problem driven.

Buerk (Buerk &Szablewski, 1993) poses the question of whether mathematics is discovered or invented to her students as part of a journal writing project. She describes the terms 'invent' and 'discover' to her students as follows:

> invent- to think up, to contrive by ingenuity, to originate something that did not exist previously (p. 159)
>
> discover- to find and bring to knowledge of the world something that has always existed (p. 159)

One of her students, Szablewski, also the co-author of the 1993 article, talks about her own reflections on the question and proposes an interesting view in her journal. She describes mathematics as a personal journey, where the mathematics that was previously invented by the mathematicians is reinvented by the students while they are attempting to understand it. She continues: "Only when we invent something for ourselves can we grasp its essence, for it is only then that we know what it is made of and how it works" (Buerk and Szablewski, 1993, p.161).

The findings of the current study indicate that most students view mathematics as discovered. This is similar to the popular view discussed in Buerk & Szablewski (1993), which they compared to the role of Christopher Columbus – 'an explorer upon the seas of education' (p. 160).

Another perspective on the 'discovered' versus 'created' issue comes from the findings of Frid and White (1995). They report that students were undecided about whether mathematics was discovered or created. Some students described mathematics as created, however, their selection was triggered by their view that created means 'made up' by mathematicians – they themselves did not see much use for mathematics and had difficulty understanding it. Frid and White's observation, as well as Szablewski's seems to suggest that students attach their own interpretation to the terms discovered and created. Furthermore, their sense of ownership of the mathematical ideas affects their views on the 'discovered' versus 'created' discussion. Thus, it is not surprising that the majority of students in this study viewed mathematics as discovered. This view tends to project the image of a typical learner whose primary concern in a mathematics classroom is to assimilate a set to tools that will carry him or her safely through a mathematics course.

Summary of Main Findings - Research Question One

Conceptions of Mathematics as a Discipline

Students seem to hold five general conceptions of mathematics listed below in descending order of frequency. Each of these conceptions has two components. The first component describes individual views of mathematics as a discipline. The second component describes views on the use of mathematics. For example, in the conception of mathematics as a 'mental activity' for developing problem solving techniques, the first component is the description of mathematics as a 'mental activity; the second component is the description of how this activity is used, e.g., for developing problem solving techniques.

- Students view mathematics as a school subject that is used in a variety of contexts – everyday life, at work, for learning other subjects and for future studies. Everyday and professional uses are more frequently reported.
- Students view mathematics as a form of mental exercise or activity that is used to develop intellectual and problem solving abilities. Intellectual ability is more frequently reported.
- Students view mathematics as a group of numbers and rules for doing calculations.
- Students view mathematics as language used to express relations between objects and quantities, and to describe the physical world. Relations between objects and quantities are more frequently reported.
- Students view mathematics as a form of art – an activity that allows us to express our creativity.

Examining the metaphors used by the students, we can see parallels between the choice of conceptions and the choice of metaphors in two ways. Some metaphors reflect the first component describing views on mathematics as a discipline. Others reflect the second component describing its uses.

Metaphors of Mathematics

- The most frequently reported metaphor of 'subject' reflects the conception of mathematics as a school subject used in a variety of contexts, which is also the most frequently reported conception.
- While none of the conceptions is directly reflected in the metaphor of 'skills', they may be related though the utility component of each conception. To illustrate: (a) mathematics as a school subject helps students develop several academic and intellectual skills, (b) mathematics as a mental exercise or activity helps students develop intellectual skills, (c) mathematics as a group of numbers and rules helps students to develop their computation skills, and (d) mathematics as a language helps students develop their communication skills.
- The metaphors of 'thing' and 'life necessity' reflect individual attitudes and value judgments about the importance or role of mathematics. They may be related to the second component in the conceptions, where students describe their views on the uses of mathematics. Something is deemed important, useful, etc. because of its potential benefit in a person's life.
- The metaphors of 'numbers and symbols' as well as 'procedures or rules' reflect the conception of mathematics as a group of numbers and rules.
- The metaphor of 'mental exercise or activity' reflects the conception of mathematics as a mental exercise or mental activity.

- The metaphor of 'language' reflects features of the conception of mathematics as a language of science.

Evolution of Mathematics

The majority of students are of the view that mathematics was discovered as people attempted to understand the world around them. Mathematics as 'discovered' may be interpreted as a product ready to be received and used. This is not surprising, if we compare this result to the findings on the conceptions of mathematics and the metaphors used to describe it. The conceptions of 'school subject' and 'numbers and rules' accounted for more than half the number of responses. These conceptions tend to project an image of mathematics as a 'finished' product. The same discussion applies to metaphors such as 'subject', 'skills', 'thing', 'life necessity' and 'numbers and symbols'.

Research Question Two

What are student conceptions of the utility of mathematics in everyday life?

Table 5.3 summarizes the findings of Research Question Two from three perspectives: uses of mathematics, most useful skills, and most useful subjects.

Table 5.3

Summary of Research Question Two results (conceptions of the utility of mathematics)

Uses	Useful Skills	Useful Topics
'day to day routines' (28%) 'calculations and estimations' (19%) 'developing intellectual skills' (14%)	Intellectual Skills (58%) thinking skills ((61%) attention/concentration (59%) solving problems (39%) memory (35%) learning (34%)	Arithmetic (82%) calculations (60%) reading numbers (46%) percentages (41%) fractions (24%) ratios (22%)
'communicating ideas related to numbers and measurement' (12%) 'advancing science and technology' (8%) 'activities such as puzzles or games' (5%)	Practical Skills (23%) everyday life (52%) calculator / computer (45%) scientific (24%) business (19%) academic (13%)	Algebra (8%) word problems (53%) equations (43%) square roots (14%) formulas (13%)
'work related tasks' (5%) 'art activities' such as drawing (3%)	Mathematical (10%) do calculations (56%) read / interpret charts (23%) measurement (23%)	Statistics (7%) charts (21%) averages (14%) probabilities (8%)
'academic & professional development' (2%) 'understanding government policies' (0%)	Communications (8%) deal with others in market (53%) communicate through numbers (26%) communicate with essence (12%) language skills (9%)	Geometry (3%) perimeters... (19%) theorems (18%) measurement (16%) basic shapes (7%)

Notes. The three perspectives: uses, useful skills and useful topics are derived from responses to the following items of the FN-Q questionnaire: FN-Q12 (uses of mathematics), FN-Q9 & FN-Q11 (useful skills) and FN-Q3 & FN-Q5 (useful topics)
The percentages represent the response frequencies rounded to the nearest whole number and arranged in descending order per item.

Uses of mathematics - Why is mathematics useful?

The majority of students view mathematics as useful for dealing with a variety of personal and professional contexts. Namely, as reported in Table 5.3, it is useful for dealing with 'day to day routines', 'work related tasks' and for 'academic and professional development'. Mathematics is less frequently viewed as useful for dealing with 'calculations and estimation', and 'communicating ideas related to number and measurement'.

Students also report that they view mathematics to be useful for 'developing intellectual skills', as reported in Table 5.3. A smaller percentage of students view it as useful for 'activities such as puzzles or games', 'art activities' and for 'advancing science and technology'. No one views it as useful for 'understanding government policies'.

In the report Everybody Counts (National Research Council, 1989), mathematics is seen as influencing our personal and work lives at five different levels: practical, civic, professional, leisure and cultural.

- Practical – knowledge used to improve basic living standards; this is an objective of universal elementary education. This notion may be related to the view in the current study that mathematics is useful for 'day to day routines', 'calculations and estimation' and 'communicating ideas relating to numbers and measurements'.
- Civic – concepts that help understand public issues; this is an objective of secondary school to create an "enlightened citizenry". In the current study, this notion was not selected by anyone.

- Professional – skills necessary for using math as a tool; this is an objective of college math courses. In the current study, this notion may be related to viewing mathematics as useful for 'advancing science and technology', for 'work related tasks', and for 'academic and professional development'.
- Leisure – disposition to enjoy math as a form of logical challenge. In the current study, this notion may be related to the view of mathematics being useful for 'activities such as solving puzzles and games'.
- Cultural – role of math as an intellectual tradition, appreciated for its power and beauty. "Like language, religion, and music, mathematics is a universal part of human culture" (p. 33). In the current study, this may be related to the use of mathematics for 'art activities', one of the less frequently reported views in Table 5.3.

The findings of the current study demonstrate that the practical and professional uses are the most frequently cited. The cultural and leisure uses are not view as equally important. The civic use was not viewed as important.

Winslow (1998) presents five arguments for the teaching of mathematics:

- Training - for logical and deductive analysis
- Prosperity – prerequisite for technological developments and financial prosperity
- Aesthetics - intellectual structure to be appreciated
- Filter – for providing educational opportunities
- Democracy – for informed citizenship

While the first three are traditional arguments, according to Winslow, the fourth is "an observation of an actual function of the mathematics discipline in many contexts, though not a function desired by its authors." (Winslow, 1998, p. 18) The fifth is a personal view of the author. Out of these, the training and prosperity arguments may apply to the uses reported in this study. The aesthetics argument may be related in part to the intellectual use. The filter argument is not reflected in the current data. As for the democracy argument, it was not deemed as useful by the sample.

Frid and White's (1995) study focuses, amongst other things, on the utility aspect of mathematics. Their findings indicate that students found mathematics to be important for everyday life. This corresponds with the highest conception of utility in this study – mathematics is useful for 'day to day routines'. Students in Frid and White's study questioned the extent of mathematics needed. Furthermore, while they view mathematics as useful for future studies or for their future careers, they have their doubts about the degree to which mathematics is used professionally. Students in the current study also reported that mathematics is important for future studies and careers, but with much less frequency than daily use. Frid and White grouped both student and teacher views on the utility of mathematics into two categories: use of basic mathematics and skills in everyday life and potential use of advanced mathematics in professional life. Teacher responses reflected a third category – use of mathematics 'as a reflection of human thinking or culture' (p. 17). This view shares common features with one of the conceptions derived in this study - the use of mathematics in 'art activities' as a form of culture. However, this conception was not frequently cited.

Grouws et al. (1996) also had a dimension of utility of mathematics in their study on student conceptions. Their results indicated that both the mathematically talented group and the regular group of high school students found mathematics to be useful in their personal lives, in school and out of school situations, and for future plans.

Uses of Mathematics - Differences in Group Responses

As reported in Chapter IV, the results indicate that Academic Level appears to be the only one variable revealing some differences in the way the different groups addressed the multiple choice item FN-Q12 relating to the uses of mathematics. For statistical purposes relating to the frequencies in the Chi-Square cells, I combined the groups with high school and post high school education.

A Chi-Squared test revealed no significant difference between the group responses to the four highest ranking conceptions of FN-Q12. Table 5.4 provides the details.

Table 5.4

Chi-Squared test for groups with different academic levels on FN-Q12 (uses of mathematics)

	Practical	Calculations	Intellectual	Communication
No High School Observed Frequency	17	7	6	7
No High School Expected Frequency	16.1	9.0	6.6	5.2
High School or more Observed Frequency	63	38	27	19
High School or more Expected Frequency	63.9	36.0	26.4	20.78
Chi-Squared Value *		1.4734		

* Critical value as calculated by EXCEL function CHIINV = 7.8147 for one tailed probability p= 0.05 with df = 3

Notes. Practical, Calculation, Intellectual and Communication are the four highest ranking conceptions of utility of mathematics. They are presented below:
Practical – mathematics is useful for dealing with day to day, work related tasks and for academic and professional development
Calculations - mathematics as useful for dealing with calculations and estimation
Intellectual – mathematics as useful for developing intellectual skills
Communications - mathematics is useful for communicating ideas related to numbers and measurements

Most useful skills - How is mathematics useful? What skills does it help us develop?

More than half the students found intellectual skills to be the most useful, as reported in Table 5.3. Among the intellectual skills, thinking skills, attention and concentration, and problem solving skills are the most frequently reported.

Nearly one quarter of the students selected practical skills as the most useful, as reported in Table 5.3. Among the practical skills, everyday skills and calculator and computer skills were the most frequently reported.

Mathematical and communication skills were less frequently cited, as listed in Table 5.3. Of mathematical skills, calculations was the most highly rated. Of communication skills, dealing with others in the market was the most highly rated.

In *Everybody Counts* (National Research Council, 1989), mathematics is seen as helping develop, among other things, critical mental abilities. The Employability Skills Profile (Conference Board of Canada, 1993) refers to using mathematics to solve problems and understand results, as well as for reading and interpreting graphs, charts and displays. Carnevale et al. (1988) mention problem identification, reasoning, estimation and problems solving, in addition to basic computation, amongst several required workplace basic computational skills.

Carenvale et al. (1990) discuss *The Conrnell Institute for Occupational Education*' list of required computational workplace skills, which includes:

- Quantification: reading, writing, counting, ordering and comparing numbers
- Computation: operations with integers, fractions and decimals

- Measurement and Estimation: measuring time, temperature, distance, length, volume, height, weight, velocity and speed, using and judging the suitability of exact measures, estimating quantities with an appropriate level of accuracy
- Problem Solving: using above mentioned skills for practical applications
- Comprehension: ability to use the knowledge and skills in the following areas: Measures; Display, collection and interpretation of data, setting up and solving algebra equations, using geometry principles and formulas

These skills are similar to the intellectual, practical, and mathematical and communication skills identified in the current study. For example, the 'read/interpret chart' listed under mathematical skills may be associated with the 'Comprehension' skill of displaying, collecting and interpreting data. Another skill that may be associated with 'Comprehension' is 'communicate though numbers', listed under communication skills. In displaying, collecting and interpreting data, we are using numbers to communicate ideas in a graphical form.

Most Useful Skills - Differences in Group Responses

To examine the differences in the responses of different groups to multiple choice item FN-Q9 (most useful skills), I used the Chi-Squared. Due to the difference in group sizes and its effect on the expected frequencies, I combined the groups with high school or post high school education. The Chi-Squared test revealed no significant difference between the group responses to the three highest ranking skills. Table 5.5 provides the details.

Table 5.5

Chi-Squared test for groups with different academic levels on FN-Q9 (most useful skills)

	Intellectual Skills	Practical Skills	Mathematical Skills
No High School Observed Frequency	21	10	2
No High School Expected Frequency	26.7	10.3	4.4
High School or more Observed Frequency	112	41	20
High School or more Expected Frequency	106.3	40.7	17.6
Chi-Squared Value *		3.2148	

* Critical value as calculated by EXCEL function CHIINV = 5.9914 for one tailed probability p= 0.05 with df = 2

Although there is no evidence of statistically significant differences amongst students with different academic levels in relation to the choice of useful skills, it is interesting to note that none of the students with post high school education regarded practical skills as important. On the other hand, they ranked mathematical skills more highly than the other two groups. They ranked intellectual skill more highly than the other two groups especially the group with no high school education.

Most useful subjects/topics - Which mathematical topics are useful?

Arithmetic is reported to be the most useful subject in mathematics. The arithmetic topic of calculating amounts is the highest on the checklist of most useful topic followed by reading and writing numbers, and working with percentages (listed in Table 5.3).

Algebra, Statistics and Geometry have much lower ratings. The most useful of the Algebra topics is solving equations. In Statistics, the most useful topic is using charts. None of the Geometry topics enjoyed a high frequency. However, the highest amongst the geometry topics is calculating area and perimeter (see Table 5.3).

In Frid and White (1995) students often indicated that the mathematical knowledge relevant to their everyday life is mostly primary school mathematics. They considered 'basics' to be the most relevant. They viewed high school mathematics as relevant for university or for future careers. Moreover, high school mathematics was seen as relevant to a limited number of careers. This view was shared by some teachers who also questioned the potential use of many topics in high school mathematics. In some ways, these views coincide with the findings reflected in the current study that tend to emphasize Arithmetic skills, skills mostly developed in primary school.

Most Useful Subject - Differences in Group Responses

There was little difference in the way the different groups rated the most useful subject. The majority of students, about 82%, ranked arithmetic as the most useful subject.

Summary of Main Findings - Research Question Two

Conceptions of Utility of Mathematics

Students reported seven general conceptions of the utility of mathematics listed below in descending order of frequency.

- Students view mathematics as useful in an individual's life in a variety of contexts: 'day to day routines', 'work related tasks' and 'academic and professional development'. 'Day to day' routines are the most frequently reported followed by 'work related tasks'.
- Students view mathematics as useful for 'calculations and estimations'.
- Students view mathematics as useful for 'developing intellectual skills'.
- Students view mathematics as useful for 'communicating ideas related to numbers and measurement'.
- Students view mathematics as useful for 'advancing science and technology'.
- Students view mathematics as useful in leisurely 'activities such as puzzles and games'.
- Students view mathematics as useful in 'art activities' such as drawing.

Most Useful Skills

Examining the most useful skills reported by students, we can see parallels between some of the conceptions of utility and the useful skills reported.

- The most frequently reported skills, the intellectual skills, are related to the conception of mathematics as useful for 'developing intellectual skills'.

- The practical skills are related to the conception of mathematics as useful for dealing with a variety of contexts e.g. 'day to day routines'.
- The mathematical skills are related to the conception of mathematics as useful for 'calculations and estimation'.
- The communication skills are related to the conception of mathematics as useful for 'communicating ideas related to numbers and measurement'.

Most Useful Topics

About 82% of the students ranked Arithmetic topics as the most useful. Algebra, Statistics and Geometry are less frequently cited (18% for all three). The emphasis on Arithmetic and its skills is also apparent in the types of examples given in response to Research Question Three, listed in Table 5.6 in the next section.

Research Question Three

What are student conceptions of the links between mathematical knowledge and its applications in the work environment?

Table 5.6 summarizes the findings on the examples of daily and work related tasks.

Table 5.6

Summary of Research Question Three results (examples on the applications of mathematics for daily and work related tasks)

Daily Task Examples	Work related Task Examples
market/shopping (28%)	accounting (28%)
time/calendar (20%)	numerical information (6%)
budget/expenses (17%)	tabular information (6%)
calculating amounts (8%)	calculating amounts (5%)
using numerical information (6%)	time/calendar, banking, statistical analysis (4% each)
shapes and measurement (3%)	problem solving, budgets/tenders (3% each)
studies, personal banking (2% each)	studies, mental ability (2%)
estimating amounts, cooking/baking, counting, mental ability, using percentages, calculator/computer, problem solving (1% or less)	file organization, academic evaluation, computer/calculator, counting objects, development, engineering drawings, trade/commerce, ordering numbers (1% each)

Note. The percentages represent the response frequencies rounded to the nearest whole number and arranged in descending order per item

Examples related to using money for shopping are the most frequently cited in the daily task list, closely followed by time and calendar use, as well as dealing with home and personal budgets and expenses (see Table 5.6). Less frequently reported examples include, amongst others, calculating amount, using numerical information as well as shapes and measurement (for a complete list, see Table 5.6). Keeping in mind that the majority of students had no work experience, their immediate exposure to mathematical notions came though school or in their daily routines which involved a lot of Arithmetic. Thus it is not surprising that Arithmetic was reported to be the most useful skill for more than 80% of the students. The type of examples listed here strongly reflect this choice such as market and everyday shopping, time and calendar, budget and expenses, calculating amounts, and using numerical information.

Some students had difficulty in responding to the item on work related tasks as indicated by the high percentage of missing responses (around 26%). The emphasis on Arithmetic can also be seen in the list of work related examples. Examples on money, as in accounting, are the most frequently cited in the work related tasks list. Using numerical and tabular information are much less frequently reported, followed by calculating amounts, time and calendar use. The work related list also includes examples such as banking, statistical analysis, budgets and tenders, problem solving and other examples

Gal and Stoudt (1997-1998) provide examples of how mathematics is used in an adult's everyday life:

> Family and Home- shopping, home repair, cooking, coordinating schedules, understanding prescription labels, personal finance, active parenting
>
> Workplace – shipping merchandise, measuring, calculating needed material, reading assembly instructions, retrieving data form a computer system, learning statistical processes control, planning timetables
>
> Community- informed citizenship (poll results, crime figures, communicating with public officials, social action (fundraising, surveys, and environmental implications)
>
> Further education- studying college level courses or taking workplace training, sitting for entrance exams or qualifying exams

Some of the examples reported in this study relate to the Gal and Stoudt's list. However, the list provided by the students misses commonly used applications such as understanding prescription labels and community applications.

Summary of Main Findings - Research Question Three

Only 20% of the sample had some type of work experience, either regular or in work placement. This is reflected in the types of examples used in the work related tasks list. Several examples are common to both lists. Very few examples address professional applications strictly related to specific work situations.

'Monetary' Applications

The most frequently cited examples in both lists share a common feature – they deal with different types of money related applications.

- In the daily tasks, market/shopping, budget/expenses and personal banking account for 45% of the types of examples listed. The missing responses account for about 8% in daily task examples. Thus 45% of the remaining 92% of responses listing monetary applications constitutes nearly half the number of examples provided.
- In the work related tasks, accounting, budgets/tenders, as well as banking account for 35% of the types mentioned. The missing responses account for about 26% in work related examples. Thus the 35% of the remaining 74% of responses listing 'financial' applications constitute about half the number of the examples provided.

Other Common Examples

There were other common examples involving the use and calculations of time - time/calendar, the classical use of calculations - calculating amounts; using numbers in a variety of ways – using numerical information. Less cited examples (3% frequency or lower) include use for problem solving, for studies, in developing mental ability, for calculator/computer applications, counting or ordering numbers.

Different Examples

- Use of tabular information appeared in the work related tasks but not the daily ones. This example was cited with a 6% frequency, the second highest on the list.
- Use of shapes and measurement appeared in the daily but not the work related list. This example was cited with a low frequency, 3%.
- One domestic application that appeared in the daily list is cooking and baking. On the other hand, specialized applications appearing in the work related list included file organization, academic evaluation and engineering drawings. However, these examples were cited at a very low frequency of around 1% each.

Overall Summary of Findings

Conceptions of Mathematics and its Utility

The findings suggest that around 86% of students participating in the current study hold the following broad conceptions of mathematics: (a) a school subject that is useful for everyday life, for work and for future studies (46%), (b) a form of mental activity that is useful for developing intellectual and problem solving abilities (31%) and (c) a group of numbers and rules for doing calculations (9%). These findings correspond with the three highest ranking conceptions of utility, which account for 68% of student responses. Students view mathematics as useful for (a) day to day routines, work related tasks, and academic and professional development (35%), (b) doing calculations and estimations (19%) and developing intellectual skills (14%).

There are two more conceptions reflected in this study's data, but less frequently cited. Mathematics is viewed as: (a) a form of language (6%) and (b) a form of art. These findings correspond with the following conceptions of utility: (a) communicating ideas related to number and measurement (12%) and (b) art activities (3%).

There seems to be a correspondence between students' views of mathematics and their views on its utility. There are two additional conceptions of utility: (a) advancing science and technology (8%) and (b) use in activities such as puzzles (5%), where the latter may be associated with using mathematics to develop intellectual abilities.

Most Useful Skills

The majority of students view intellectual skills (e.g. thinking) and practical skills (e.g. everyday life skills) as the most useful skills that mathematics helps them develop. Together, they account for 81% of the student responses, with intellectual skills coming first (58%). Students also cite Mathematical skills (calculations) as useful but with a much lesser frequency (10%). Comparing these with the views on conceptions of mathematics and its utility, we can see a correspondence in the views but not in their ratings – practical skills are not the first choice, as is the case with the views on conceptions. Rather, students view intellectual skills as the most important.

Communication skills (e.g. dealing with others in the market), the least useful skill cited (8%), correspond with the conception of mathematics as a language. They both share the least ratings.

Most Useful Topics

Arithmetic is viewed as the most useful skill by 82% of the students. The most important arithmetic topic listed is calculations. This choice of subject has some correspondence with the conception of mathematics as a school subject used in a variety of contexts, and as a group of numbers and rules for doing calculations. Arithmetic is the most commonly used subject in mathematics in a variety of contexts, especially calculations. The other subjects: Algebra (8%), Statistics (7%) and Geometry (3%) where much less frequently cited.

Types of Examples

The most frequently cited examples for daily and work related use were related to monetary applications such as shopping and expenses. Other common examples included dealing with time, using numerical information and for calculation purposes. There were very few specialized applications (e.g. engineering drawings, file management).

Interpretation of Findings

Our conceptions of mathematics are the product of societal factors, as well as our own personal experiences in and out of school. These conceptions are influenced by (1) parents, family, and societal views of mathematics and by its value, (2) teachers and school environment and (3) our own 'mathematical' learning experiences – the triumphs and frustrations that we encounter along the way.

In the current study, I chose to focus on identifying and describing the conceptions themselves as a starting point. However, to be able to understand the conceptions, we need to keep in mind the factors that may have influenced them. The findings from this study suggest that the most commonly held conceptions of mathematics and its utility is that of a school subject that can be used for several practical purposes. These conceptions correspond with other findings in the study. The highest ranking metaphor used by students is also 'school subject'. The most useful subject reported by students is Arithmetic which tends to have many practical applications in a variety of contexts. The types of examples of the uses of mathematics for everyday life tasks and for work, also correspond to the choice of arithmetic as the most useful subject. They tend to emphasize calculations related to money, and in some cases to time. However, the most useful skill that mathematics helps us develop, according to the findings, is intellectual skills. Practical skills are listed as the second most important. This view may have been influenced by the popular views on mathematics as being for the 'intellectuals'. How often do we hear statements such as: "Mathematics is hard. Only a genius or a 'math-brain' can understand it" or "One must have a mathematical mind to do math" (Zaslavsky, 1994, p. 21 and 48). Furthermore, most students in the current study view mathematics as discovered (64%) as opposed to created. This view along with the conceptions presented in the previous paragraph suggest the conception of mathematics as a finished product with fixed content to be delivered and applied when needed.

The evolving general conception of mathematics derived from the current data, is not untypical. More often than not, mathematics is presented as a school subject of value later in life. It is 'transmitted' to the students though teacher explanations followed by practicing exercises and solving problems that supposedly constitute real life situations. The typical students assume a passive view role in the classroom. They tend to become recipients rather than participants, accepting the fact that mathematics is useful and that they need to study it and get passing grades.

When students inquire about the potential value of mathematics, the typical answer received is that mathematics is useful for everyday life, work and future studies. Some teachers emphasize its role in developing intellectual skills. Parents stress good performance in mathematics because it is the key to 'profitable' or 'prestigious' professions such as medicine, engineering and accounting. Society emphasizes the value of mathematics and its place as an 'intellectual' discipline – only 'smart' people can do mathematics.

While several students in this study view mathematics as useful for everyday life, work and future studies, their examples indicate a limited scope of application. The most frequently cited examples concentrate mostly around dealing with money, time, calculating amounts, as well as using numerical information (e.g. for post box, for numbering files, etc.). This may be a reflection of the types of 'applications' included in the mathematics textbooks and emphasized in the classroom. A closer look at the examples provided demonstrates that there are many vital applications missing such as health sciences, public policies, fine arts, scientific applications, astronomy, social sciences and others. Furthermore, most examples given tend to be of a general nature e.g. mathematics is useful for shopping, for calculating time, for banking, etc. Very few examples provided a link between the topic and the application. Basically, most examples revolved around numbers and calculations.

Another general conception of mathematics that may be derived from the current data is mathematics as a mental activity that is used to develop intellectual abilities, including problem solving. This notion can also be observed in the conceptions of utility of mathematics where students view it as useful for developing intellectual skills, and for activities such as puzzles or games. This view of mathematics, although less frequently cited, is not uncommon. Amongst the many who do not, there are always some students who view mathematics as an intellectual activity – a challenge. Citing intellectual skills as the most useful skill complements this conception. Although not statistically significant, it is worthwhile to note the differences in which the groups with different academic levels viewed this conception. The students with post high school education tended to cite this conception more frequently than the other two groups. Students with high school education or less tended to view mathematics in light of its practical aspects.

Although some students hold the conception of mathematics as an intellectual exercise or activity, and several students view mathematics as useful for developing intellectual abilities, the types of examples and topics listed do not seem to reflect this view. Very few examples dealt with 'intellectual applications'. When they did, it was at the general level e.g. problem solving.

In the last chapter, I will discuss the study implications from three perspectives: researcher, instructional designer and teacher. I will suggest potential benefits in the form of recommendations. Next, I will discuss the limitations of the study. Finally, I will conclude by discussing future directions of the research.

CHAPTER VI

CONCLUSIONS AND FUTURE DIRECTIONS

A Researcher's Perspective

The research involved an iterative process with three interrelated studies. I started the first study with an exploratory questionnaire using open ended items. A procedure built into open-ended questionnaire items used writing as a tool to explore student conceptions of mathematics. This procedure proved valuable in collecting a considerable amount of data that allowed me to derive categories of conceptions of mathematics and its utility. In the current literature on mathematics education, the use of student writing in the learning process has received considerable attention (Shield & Galbraith, 1998). In writing, students express their thoughts, feelings and experiences. Standera (1994) suggests the use of writing as a tool for understanding how students feel about mathematics, how they solve problems and how they view themselves as learners. She emphasizes writing as a form of communication: "Writing establishes an excellent line of communication between me and my students. It gives me some valuable information about all my students, not just the ones that easily share their thoughts and ideas in conversation" (p. 31).

The results of the first study were used to construct a second questionnaire that was tested in the pilot study. The categories of conceptions were revised and so were the questionnaire items. After several modifications based on the results of the pilot study and formative evaluation procedures, the final questionnaire was constructed. The iterative process aimed at preserving as much as possible the validity of the originally derived conceptions. I introduced some modifications when deemed necessary. However, these modifications were mainly in language and structure. I did not use the literature on conceptions as my starting point primarily because I wanted the conceptions to evolve out of the participants' own views. However, literature came in at the second stage where I added a few categories such as mathematics as a form of art.

In the final questionnaire, I included open ended, checklist and multiple choice items. Each type served a different purpose. The multiple choice items, derived from the categories of conceptions established in the first two studies, were intended to identify student views on (a) the essence of mathematics (b) the evolution of mathematics (c) the utility of mathematics (d) the most useful skill that mathematics helps us develop; and (e) the most useful mathematical topics. The students were instructed to make one selection only. Responses to these items were used to describe the categories of broad conceptions held by students.

One of the open ended items was about metaphors. It gave students the chance to describe what mathematics meant to them in their own words. This item complemented the items on conceptions of mathematics. The two checklist items complemented the items on most useful skill and topic, by providing the students with a list of examples of skills or topics derived from the first two studies. There was one more open ended item that required providing examples of how mathematics is used in everyday life and at work. It complemented the items on the utility of mathematics by identifying concrete situations where mathematics is seen as useful.

An Instructional Designer's Perspective

A key aspect of any instructional program is its short-term and long-term benefits. When the program is over, can learners demonstrate an understanding of the concepts as they relate to their needs and uses? Could learners recognize situations where those concepts are needed, and can they use them in solving relevant problems? Could learners recognize when and how to apply the skills learnt?

The type of skills, mathematical and other, needed in today's society is changing. *The National Council of Teachers of Mathematics* in a re-examination of the goals of education focuses on the need for change in the educational system to meet the economic needs of this information age:

> All industrial countries have experienced a shift from an industrial to an information society, a shift that has transformed both the aspects of mathematics that need to be transmitted to students and the concepts and procedures they must master if they are to be self-fulfilled productive citizens in the next century (The National Council of Teachers of Mathematics, p. 3).

These changes pose a great challenge to people working in the field of educational technology in general and instructional designers in particular. How can we help learners become 'self fulfilled productive citizens'? How can we help them see the links between what they need and use? How can we help them make their learning experiences more meaningful and more lasting? We could start with a closer look at our learners as individuals. We are aware of the importance of knowing how to address learners to make instruction more relevant and meaningful. Understanding learners and trying to see things through their eyes provides us with a sense of direction that can guide us through instructional design process.

One of the most challenging tasks encountered by an instructional designer is visualizing their target audience. Who are those learners? What do we know about them as individuals i.e. aspirations and ambitions, beliefs and conceptions, motivation to learn - those attributes referred to as the affective domain? There are three groups of learner characteristics that are considered critical to the instructional design process depending on the task at hand: cognitive, psychosocial and physiological (Smith and Ragan, 1993). The psychosocial characteristics are very diverse and include amongst other things, interest, motivation, attitudes, and beliefs. Main (1993) emphasizes the importance of the affective domain in the instructional design process. He quotes some as suggesting that curiosity, interest and motivation gradually diminish in students because of the instructional procedures used in classrooms. Many instructional programs fail because of lack of concern for the motivational side (Spitzer, 1996). Mathematics programs are no exception.

As instructional designers, we face the difficult task of making assumptions about the learning environment and the learners. Some assumptions are based on facts; others are subjective and based on our experience. Often, we have a hard time assuming anything, especially if the only knowledge we have is the course description or outline. As teachers we encounter similar problems. We know what the course is about; we know the content and scope, etc. But, what do we know about the people involved in the teaching learning process, the learners? The more informed we are, the better assumptions we make. These assumptions have a bearing on our choice of instructional strategies, learning activities, and evaluation procedures. It is hoped that the results of this research will allow us to make more informed assumptions and consequently more suitable decisions regarding the instructional design process by adding to our knowledge of learner characteristics and guiding us in selecting suitable instructional strategies and resources. I will elaborate on these ideas in the next three sections.

Learner Profiles

Within the individual or group profiles describing learner characteristics, it may be helpful to add a component for conceptions of the subject matter, in this case - mathematics. One of the many things that we can attempt to know about our learners is their views, how they perceive things. This would help us communicate with them more effectively. One possible assumption that could be made, in the case when students think of mathematics as a set of rules to be memorized and applied, is that they will most likely adopt a passive attitude in class. They will tend to be passive recipients rather than active participants in a classroom. They will expect to be provided with these rules and trained to apply them successfully.

Instructional Strategies

Our knowledge of the learner views on mathematics can guide us in the selection of instructional strategies and learning activities that are more suitable to learner needs; and that would allow them to expand their knowledge and abilities as they progress in the course. Taking the example mentioned in the section on learner profiles, a student with a passive view of mathematics may have a hard time dealing with situations that require independent thinking. He or she may need learning activities which provide guidance in the form of leading questions, or other, that would help them gradually proceed to a more independent level.

Content and Applications

Our knowledge of student conceptions of the utility of mathematics may guide us in selecting activities that would allow them to experience mathematics in a variety of contexts. Looking at the results of this study, we notice that most of the examples used focused on business or trade and dealt with money and calculations. Also, if we look at the uses, not one participant viewed mathematics as helpful in understanding public policies. Yet, everyday in the news we are faced with the task of comprehending government policies on taxes, on healthcare, on social security, etc. Student conceptions are the product of several factors, one of them is their learning experiences. We need to work on broadening those experiences by integrating into our courses and resources a wider spectrum of applications that would help students see the links between mathematics and real life. Our knowledge developed out of our needs and living experiences; why not integrate these experiences into our instruction?

A Mathematics Teacher's Perspective

Almost always when introducing a lesson on solving word problems, one or more students demand to be given the 'best way' for knowing which operation to use for solving the problem. Incidents like this made me wonder why it is that after spending so much time discussing problem solving techniques and exploring possible approaches, the discussion goes back to demanding a 'magic' rule that solves problems. This scenario is not exclusive to word problems, or to these particular students. It is often repeated in other mathematics classrooms and with other topics. Students are constantly on the look for neat recipes that ensure success in mathematics.

Is mathematics such a difficult subject that the best way to survive mathematics courses is to learn a few rules and tricks that guarantee a passing grade? Are we as teachers not doing a good job explaining things to some students? Are some students incapable of learning mathematics or lacking in basic skills needed to build more advanced concepts? Are mathematical concepts, as introduced in schools or colleges, so detached from everyday life that they make little sense to the students?

"Currently, mathematics is taught as if it were a universe closed in itself, conceived, and for some even received, in the supposedly final form in which it is known" (D' Ambrosio, 1993, p.35). Fasheh (1997) goes on to say: "... the math that we teach or study has lost its life, its soul, and its connectedness to the realities in both the immediate and the wider world". He continues that we cannot keep teaching mathematics as a distant, neutral subject disengaged from the realities in which we live. Furthermore, we cannot expect to achieve relevance through superficial examples, referred to as 'applications'. He is of the view that neglecting nature, experience and reality has largely contributed to the problems encountered in the teaching and learning of mathematics.

Many students do not see mathematics around them and cannot appreciate the need to study it. They haven't been exposed to mathematics as a dynamic discipline with roots extending to many other branches of knowledge. They haven't been exposed to mathematics applied in a variety of everyday life contexts - from the objects of nature that are represented through mathematical figures, to the concepts of measurement which govern our everyday lives, to the more advanced concepts that help us better understand our world.

This brings me to the question that triggered my interest in this research. Maybe we, the teachers, and the students are having some sort of communication problems? In everyday language, maybe we are using different 'wavelengths'? To be able to help our students, we need to communicate with them effectively; we need to understand their perspectives. We need to try to see things through their eyes, be aware of how our students view mathematics. These views have an impact on how they learn and understand mathematics. According to Oaks (1994), helping students requires that we understand their conceptions of mathematics. "Because students perceive mathematics through the filter of their conception, we must work to expand their mathematical world and to help them internalize the broader messages of mathematics" (Oaks, 1994, p. 42).

Changing the way students approach mathematics requires making sure that they are in an environment that does not allow them to fall back into the process of recording and memorizing. "The traditional process of 'teacher telling, student listening' has likely helped create the rote conceptions many of our students hold and has forced them to be passive observers in the classroom" (Oaks, 1994, p.42).

The study, while exploratory in nature, provides some interesting and hopefully useful findings. What do these findings mean to me as a mathematics teacher? How will they help me improve the communication process that goes on in my classroom? How can I help my students have meaningful learning experiences in a mathematics classroom? How can I help them develop broader conceptions of mathematics? These are very ambitious questions, and many are beyond the scope of the current study. However, based on the findings, the following recommendations may prove helpful: (a) the need for a more dynamic classroom environment, (b) the use of a wide variety of meaningful contexts, (c) introducing 'real life' learning activities, and (d) presenting students with different 'faces' of mathematics. I will elaborate on these recommendations in the next four sections.

A Dynamic Classroom Environment

We need to provide students with the chance to explore and discuss their conceptions of mathematics. This can be done through student journals, class discussions, etc. It would serve two purposes: (a) help the teacher better understand his or her students view, conceptions and misconceptions, (b) help students become aware of their own conceptions and reflect on them.

We need to give students time to explore and discuss their own interpretations of mathematical ideas. We need to encourage them to become active participants in class, and to let them know that their views are important.

Wide Variety of Contexts

We need to provide wider exposure to a variety of contexts where mathematics is used in our daily and professional life – from simple things such as calculating the duration of time, reading dietary information and prescription labels, to the more complex such as reading and interpreting numerical information on public issues, interpreting statistical information and other professional applications.

'Real life' Learning Activities

We need to design learning activities where mathematical problem solving activities simulate, as closely as possible, real life situations. This is particularly important when solving problems that are supposed to be applications of real life situations but that ignore the realistic constraints.

Other 'Faces' of Mathematics

We need to bring into the classroom different faces of mathematics. We need to present our students with activities that allow them to explore: (1) the history and evolution of mathematics and its connectedness to other aspects of the human culture such as language, philosophy, science, music and arts, (2) the aesthetic features of mathematics as reflected in designs and patterns, and (3) the leisurely aspects of mathematics in the form of games and puzzles that challenge and entertain us, as well as help us expand our intellectual abilities.

Limitations

The findings of this research apply to the population from which this sample is drawn. The nature of the study, the procedures and the types of instruments used do not allow for generalizability of findings to other populations. The sample used has very particular characteristics. The participants are female college students with a uniform ethnic background, speaking the same language and having a distinctive Middle Eastern culture. This culture maintains its religious and social traditions while reaching out to embrace the new technological changes. It is a culture that encourages learning and acquiring higher education. It encourages the participation of women in public, military, health, business, education, communication, fine arts and other professional designations. The students are enrolled in publicly funded college considered to be one of the most prestigious in the country. The tuition is free and the books are provided free of charge. This makes it accessible to students with different socio-economic status.

Most of the students are graduates of public schools, where the language of instruction is Arabic.

While it is recommended to use the research procedures for other studies involving different populations, care should be taken in using the final questionnaire as an instrument. If used for exploratory purposes, it may need minor modifications based on the context. However, it is not recommended to use it for studies that seek to generalize their findings due to issues of internal consistency (Refer to *Chapter IV* for details).

Three factors may have had an impact on the internal consistency of some items: nature of data, duration and number of items.

<u>*Nature of Data*</u>

Due to the categorical nature of the data, the method chosen for establishing the internal consistency of the questionnaire is a modified version of the 'split-half' technique. As this questionnaire included three different types of items: multiple choice, checklists and open-ended, it was not feasible to split the whole questionnaire in half and find matching items to compare. Instead, I chose to construct a matching item for each of the multiple choice items, and to check the internal consistency for each pair (item and matching item). These pairs had multiple choice statements that should also be matching. This process proved to be quite challenging, as I had to make sure not to produce redundant items. Furthermore, the multiple choice statements derived per item corresponded to categories already established and carefully worded in the first two studies. I opted for using examples for matching items to avoid any change in the categories that may influence their authenticity. However, this procedure also had its challenge, namely coming up with examples that best represent these categories. The choice of example may have affected the participants' responses, as they may not have found them meaningful or relevant. Furthermore, for items FN-Q9 (the most useful skills) and FN-Q12 (uses of mathematics) the choice of examples proved particularly difficult as some examples overlap across more than one category.

Duration

On average, the questionnaire took between 25 and 30 minutes to fill, which some may have found long. People tend to lose their concentration and their interest with the passage of time, which in turn affects the way they respond to items that may require particular attention.

Number of Items

Number of items refers to the number of multiple choices provided for a particular item. Two of the five pairs of items had 8 or more multiple choices, these being FN-Q7 (nature of mathematics) and FN-Q12 (uses of mathematics). This may have made the selection process more difficult.

Contribution to Knowledge

The literature on student conceptions suggests that they have an impact on how students understand and learn mathematics. While this literature is not extensive, it is multi-faceted. Researchers define conceptions in different ways, use different instruments and come up with a variety of results based on their initial assumptions. There is need to integrate these findings in meaningful ways. Before exploring the impact student conceptions have on their learning, we need to work on a clear definition of conceptions, on the factors that may influence them and on the most suitable means for exploring them.

The current study suggests a methodology for developing instruments that explore and describe student conceptions of mathematics. Rather than a theory based approach, the questionnaire development follows an inductive method. The iterative process employed in the study provides the means to preserve the authenticity of categories of conceptions derived in the first stage, as much as possible. The items generated from the different components of the research questions allow for more comprehensive descriptions of conceptions. For example, in Research Question 1, there are three components: essence, metaphor and evolution. Students are asked to identify their conceptions of mathematics, provide a metaphor that describes their views and select a category of evolution – created versus discovered. In addition to describing student conceptions of mathematics as a discipline, the questionnaire specifically addresses conceptions of the utility of mathematics – the why, what and how views on the uses of mathematics.

While the final questionnaire has limitations that affect the generalizability of its findings to other samples (see the previous section titled *Limitations*), the procedures may be duplicated and modified based on different contexts in which they will be used. The next section, *Future Directions,* presents some suggested modifications to improve the effectiveness of the questionnaire.

In retrospect, if I were to conduct this study again, I would introduce the following modifications to monitor the internal consistency of the items through comparing the individual responses, and to provide participants with the means for selecting multiple perspectives:

1. Delete the matching items added in the final study. Instead, I would include two open ended items, similar to items EX-Q1 and EX-Q2, in addition to the existing multiple choice items.
2. Allow for more than one choice in the multiple choice items.

Future Directions

The current research experience has led me to think of a few suggestions for improving the study procedures if it is to be duplicated:

1. Conduct the exploratory studies with samples that have balanced distribution of age groups, academic levels and work experiences. This would allow for a more comprehensive examination of the relation of these variables to student conceptions, especially academic levels, although they did not reflect statistically significant differences in the current study.
2. Introduce other variables that were not explored in this study that may have an impact on student conceptions, such as gender, socio-economic status, and type of work experience, program of study or mathematical achievement.
3. Conduct the exploratory study with a larger and more diverse sample e.g. from different fields of specialization, to allow for deriving more comprehensive categories of conceptions.

4. Use a variety of data collection techniques for both exploratory and pilot studies, such as interviews, focus groups or field observations. This would serve to provide more comprehensive descriptions of the categories of conceptions.
5. Conduct the pilot study with a larger sample to allow for more than one try until the final questionnaire items can be refined to include multiple choice statements that are more representative examples for the matching items.
6. Although this research was done with college students of a particular culture, it may be worthwhile to develop similar instruments for use at different academic levels and across different cultures.
7. If the research is conducted with students currently enrolled in a mathematics course, as is the case in this study, it may be a good idea to provide them with the exploratory questionnaire in the beginning of the course, and give them a suitable duration of time to record their answers in a journal that they can go back to and reflect upon. Conceptions are critical issues that need a lot of thinking and reflection. In general, students are rarely asked to answer questions of this type. Several of my students commented that they had difficulty answering the first exploratory questionnaire because it was very 'philosophical' in nature and needed a lot of thinking. They added that this was the first time that they were asked what they think about mathematics.

The findings of this research may serve as the starting point for a series of studies conducted with the purpose of providing guidelines for the design of learning resources for workplace mathematics. Future studies should focus on the mathematical skills needed and the context in which they are used in the workplace. The following diagram describes possible studies and their focus:

First Stage Learner Conceptions (current study and ongoing)	Second Stage Content	Third Stage Context
Exploring learner conceptions of mathematics and its uses in everyday life and at the work place	Identifying mathematical skills needed for everyday life and for the workplace	Identifying realistic contexts in which mathematical skills are used in everyday life and in the workplace

It is hoped that a more comprehensive knowledge of the learners' conceptions, the content and the context, will facilitate the design of learning programs and resources that:

1. Provide realistic settings for learning mathematical skills
2. Address contextual situations that are meaningful to the learners
3. Facilitate the application of mathematical skills to different situations
4. Raise the awareness of the power and utility of mathematics in a variety of contexts

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APPENDIX A

The Exploratory Questionnaire EX-Q

Instructions

Please read the instructions carefully:

The questionnaire has two sections (A) & (B).

Section (A) includes general information about you.

Section (B) includes questions about your own views of mathematics. The questionnaire is strictly confidential. Please feel free to say what you really think. We are interested in hearing your own ideas.

Your cooperation will help us improve the quality of instruction and the learning resources used in mathematics courses.

Your views are very important to us!

Thank you for taking the time to complete
the questionnaire.

Section (A)

Provide your response in the indicated space. When appropriate, check the correct response by using √ in the corresponding box.

Date of Birth: ☐ Day ☐ Month ☐ Year

Gender: ☐ Male ☐ Female

Highest Academic Degree: (received to-date)
☐ High School
☐ Other, please specify

____

Current Program of Study _______________

Year
☐ Foundations
☐ Year One
☐ Year Two
☐ Year Three
☐ Other, please specify

____

Favorite Subjects _______________

Hobbies _______________

If you have held a job before, please complete this section, otherwise go to Section B.

Current job: _______________

Previous employment: (list the last two jobs) _______________

____

Section (B)

Write a few lines in response to each of the following questions: (If you need more space, use the back of the paper)

EX-Q1: If we were to describe language, we could say that it is a tool of communication, a means of expressing ourselves, etc. **If you were to describe mathematics, what would you say?**

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EX-Q2: We find language **useful** because it helps us know and understand others better. It also helps us express our thoughts. **Do you find mathematics useful? Why?**

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EX-Q3: In language, we study about sentence structure because it is **useful** for forming sentences, about adjectives because they are **useful** for describing things, etc. **Can you think of lessons that you found useful in mathematics? If so, what were those lessons about?**

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EX-Q4: Language helps us develop communication skills such as reading, listening, and speaking. **Did mathematics help you develop any skills? If so, what are those skills?**

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EX-Q5: Educators use number operations, averages and percentages to come up with student grades. **In your own profession, do you use mathematics**? **If so, could you describe how you use it?**

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EX-Q6: Choose a profession, **other than your own**, where you think that mathematics is used. Could you describe how mathematics is used in this profession?

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EX-Q7: Complete the following statement as in the example: To me language is like a key. It opens the door of communication. **To me mathematics is like a**

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EX-Q8: Is there any question that you think we should have asked to get to know more about your views of mathematics?

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APPENDIX B

The Pilot Questionnaire IN-Q

Instructions

This questionnaire is part of a research done on student views of mathematics. The questionnaire is strictly confidential. Please feel free to say what you really think. We are interested in hearing your own ideas.

It is hoped that the findings of this research will help improve the quality of instruction and the learning resources used in mathematics courses.

In this questionnaire, you will be completing two sections:

Section (A) includes general information about you.

Section (B) includes questions about your views on mathematics and its applications.

Your views are very important to us!

Thank you for taking the time to complete this questionnaire.

Section (A)

Provide your response in the indicated space. Where appropriate, check the correct response by using ✓ in the corresponding box.

Date of Birth

Day Month Year

Gender ☐ Male ☐ Female

Highest Academic Degree ☐ High School ☐ Other ___________

Current Program of Study ______________________________

☐ Foundation ☐ Year One ☐ Year Two ☐ Year Three

If you have held a job before, please complete this section. Otherwise, go to Section B.

Current job: ______________________________

Previous employment: (list the last two jobs) ____________________

____

Section (B)

IN-Q1: Select the statement that best describes your view of how mathematics developed.

Mathematics was

- ☐ created by people to address their different needs.
- ☐ discovered by people while attempting to understand the world around them.

IN-Q2: Select the statement that best describes your views of mathematics. Select <u>only one</u> statement.

Mathematics is

- ☐ a group of numbers and rules for doing calculations and arithmetic problems
- ☐ a subject used in the study of other subjects, in everyday life and in the workplace
- ☐ a symbolic language that expresses relations between shapes and measures
- ☐ the language of science used to describe the physical world
- ☐ a mental activity that helps develop several intellectual abilities
- ☐ a form of art that enhances our creativity and imagination

Mathematics is

- ☐ a cultural tool that promotes our understanding of our world
- ☐ a calculation tool used to do calculations in a variety of contexts
- ☐ a practical tool used to deal with everyday life and work related situations
- ☐ a symbolic tool used for expressing relations between shapes and measures
- ☐ a language tool used to communicate scientific ideas
- ☐ an intellectual tool used to develop intellectual skills, including problem solving

IN-Q3: Select the statement that best describe what mathematics is to you. Complete this statement with a suitable word or a phrase.

To me mathematics is

- ☐ a collection of ____________________
- ☐ a key to ____________________
- ☐ a mean for ____________________
- ☐ a tool for ____________________
- ☐ a language for ____________________
- ☐ (other)____________________

IN-Q4: For each statement, select the choice that reflects your agreement or disagreement with the given statement. Select only one choice

Mathematics is a cold subject with no place for emotions.

Strongly Agree Agree Undecided Disagree Strongly Disagree

Mathematics is a *cut and dried* subject with no place for creativity.

Strongly Agree Agree Undecided Disagree Strongly Disagree

Mathematics is a rational subject with no place for intuition.

Strongly Agree Agree Undecided Disagree Strongly Disagree

Mathematics is an abstract subject with little relation to the real world.

Strongly Agree Agree Undecided Disagree Strongly Disagree

Mathematics is an accurate subject, where all questions have answers that are either correct or incorrect.

Strongly Agree Agree Undecided Disagree Strongly Disagree

Mathematics is universal subject that is the same all over the world.

Strongly Agree Agree Undecided Disagree Strongly Disagree

Mathematics content is fixed. We already know all the mathematics that we need.

Strongly Agree Agree Undecided Disagree Strongly Disagree

IN-Q5: Select the statement(s) that best describes your views about the utility of mathematics. You may select more than one statement.

I find mathematics useful for

- ☐ dealing with a variety of everyday life situations
- ☐ dealing with quantities and measurements
- ☐ developing intellectual skills such as analysis, critical thinking, etc.
- ☐ dealing with a variety of work related situations
- ☐ entertainment activities such as puzzles and games
- ☐ understanding public policies related to budgets, health care system, etc.
- ☐ artistic activities such as drawing and designing, sewing and weaving, etc.
- ☐ understanding other academic subjects

IN-Q6: Arrange the mathematical subjects in order of importance. The subject that you think is most useful should be ranked as (1).

- ☐ Arithmetic
- ☐ Algebra
- ☐ Calculus
- ☐ Statistics
- ☐ Geometry
- ☐ Trigonometry

IN-Q7: Arrange the following lessons in order of importance. The lesson that you think is most useful should be ranked as (1).

In Arithmetic

- ☐ The Four Basic Operations
- ☐ Ratio and Proportion
- ☐ Reading and Writing Numbers
- ☐ Fractions
- ☐ Percentages and Applications
- ☐ Ordering and Comparing Numbers

In Geometry

- ☐ Measurements & Estimation
- ☐ Shapes
- ☐ Calculating length, area, etc.
- ☐ Space Geometry
- ☐ Theorems and Proofs

In Algebra

- ☐ Algebraic Operations
- ☐ Word Problems
- ☐ Variables and Symbols
- ☐ Radicals (e.g. Square Root)
- ☐ Formulas
- ☐ Equations

In Statistics

- ☐ Charts (Line, Pie, Bar, etc.)
- ☐ Measures of Average
- ☐ Probability
- ☐ Displaying, Organizing and Interpreting data.
- ☐ Standardized Scores (e.g. z-score)

IN-Q8: Select the type of skills that mathematics helped you develop. Then, select the specific skills. You may select <u>more than one</u> type or specific skill.

☐ **Intellectual Skills such as**

- ☐ learning and understanding
- ☐ thinking
- ☐ problem solving
- ☐ attention and concentration
- ☐ memory

☐ **Communication Skills such as**

- ☐ language skills
- ☐ communicating with numbers
- ☐ dealing with others in shopping markets
- ☐ communicating with nature

☐ **Mathematical Skills such as**	☐ **Practical Skills such as**
☐ statistical skills such as reading and interpreting charts ☐ numerical skills such as calculating and estimating ☐ algebraic skills such as solving equations ☐ geometry skills such as drawing and measuring ☐ arithmetic skills such as dealing with percentages	☐ academic skills for learning other subjects ☐ professional business skills for accounting, commerce, ... ☐ professional scientific skills for engineering design, medication administration, ... ☐ statistical skills for learner assessment ☐ everyday life skills for shopping, personal banking, ... ☐ technological skills for computer and calculator use, ...

IN-Q9: Can you think of any profession where mathematics is needed and used? In what ways?

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IN-Q10: Are there any views that you would like to express regarding the role of mathematics in our everyday lives and professions?

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About the Questionnaire

F1: Was the time given to answer this questionnaire adequate?

____

____

____

F2: Was the font used clear?

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F3: Was there enough space to write your response?

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F4: Would you have liked to have space to write your own response if it did not correspond to any of the options?

____

____

____

F5: Did you have any difficulty reading or understanding the instructions on this questionnaire? Can you please explain what those difficulties were?

____

____

____

APPENDIX C

The Final Questionnaire FN-Q

INSTRUCTIONS

This questionnaire is part of a research project on student views of mathematics. It is hoped that the findings of this research will help improve the quality of instruction, and the learning resources used in mathematics courses. The questionnaire is strictly confidential. Please feel free to write what you really think. We are interested in your own ideas.

In this questionnaire, you will be completing two sections:
SECTION (A) includes questions about your views on mathematics and its applications.
SECTION (B) includes general information about you.

Your views are very important to us! Thank you for taking the time to complete the questionnaire.

SECTION (A)

FN-Q1: Select the statement that best describes how mathematics helps you. Select <u>only one</u> statement.

I find mathematics helpful in

- ☐ telling the time
- ☐ finding out how much money is left after shopping
- ☐ solving puzzles in magazines or newspapers
- ☐ dealing with difficult situations that require analysis
- ☐ designing patterns for weaving
- ☐ understanding how the social security system works
- ☐ taking courses to improve some computer skills
- ☐ reading tasks and duties schedule
- ☐ finding new ways to improve our world
- ☐ describing distances between places

FN-Q2: Select the most useful skill that mathematics helps you develop. Select <u>only one</u> skill.

- ☐ learning and understanding
- ☐ communicating using numbers
- ☐ knowing time differences between countries
- ☐ solving equations

FN-Q3: Select the mathematical subject that you use most. Select <u>only one</u> subject.

☐ Arithmetic ☐ Geometry ☐ Algebra ☐ Statistics

FN-Q4: Complete this statement with a phrase that expresses how you think of mathematics. To me mathematics is like a (n) ______________________

____

FN-Q5: Select the mathematical topic that you think is most useful. You may select <u>more than one</u> topic.

- ☐ Using Charts
- ☐ Calculating Amounts
- ☐ Reading and Writing Numbers
- ☐ Working with Fractions
- ☐ Calculating Area, Perimeter, etc.
- ☐ Working with Percentages
- ☐ Comparing and Ordering Numbers
- ☐ Using Formulas to Calculate Quantities
- ☐ Using Probabilities
- ☐ Solving Equations
- ☐ Estimating Measurements
- ☐ Recognizing Basic Shapes
- ☐ Working with Ratios
- ☐ Working with Averages
- ☐ Working with Square Roots
- ☐ Knowing Theorems and Proofs
- ☐ Solving Word Problems

FN-Q6: Select the statement that best describes your view of how mathematics developed. Select <u>only one</u> statement.

- ☐ People created mathematics to address their practical needs.
- ☐ People discovered mathematics as they attempted to understand the world around them.

FN-Q7: Select the statement that best describes your view of mathematics and its applications. Select <u>only one</u> statement.

I think of mathematics as

- ☐ a group of numbers and rules for doing calculations
- ☐ a school subject used in learning other subjects and for future studies
- ☐ a subject used in everyday life and at work
- ☐ a symbolic language that expresses relations between objects and quantities
- ☐ a language of science used to describe the physical world
- ☐ a mental exercise that helps develop intellectual abilities
- ☐ an activity that allows us to express our creativity
- ☐ a mental activity used to develop problem solving techniques

FN-Q8: Select the mathematical topic that you use most. Select <u>only one</u> topic.

- ☐ numbers and calculation
- ☐ solving equations
- ☐ shapes and measurement
- ☐ averages and probabilities

FN-Q9: Select the most useful skill that mathematics helps you develop. Select <u>only one</u> skill.

- ☐ Communication
- ☐ Intellectual
- ☐ Mathematical
- ☐ Practical

FN-Q10: Select the statement that best describes your view of mathematics and its applications. Select <u>only one</u> statement.

I think that mathematics is a tool for

- ☐ explaining things such as rules of gravity
- ☐ dealing with numbers
- ☐ dealing with day to day and work related situations
- ☐ developing our ability to think
- ☐ studying other subjects
- ☐ developing problem solving techniques
- ☐ representing relations between objects and quantities
- ☐ enhancing innovations

FN-Q11: Select the most useful skill that mathematics helps you develop. You may select <u>more than one</u> skill.

- ☐ reading and interpreting charts
- ☐ dealing with others in markets
- ☐ communicating with nature
- ☐ measuring
- ☐ doing calculations
- ☐ solving equations
- ☐ solving problems
- ☐ attention and concentration
- ☐ communicating through numbers
- ☐ thinking skills
- ☐ learning skills
- ☐ academic skills for learning other subjects
- ☐ language skills
- ☐ business skills
- ☐ scientific skills
- ☐ calculator and computer skills
- ☐ memory skills
- ☐ everyday life skills

FN-Q12: Select the statement that best describes your view on the usefulness of mathematics. Select <u>only one</u> statement.

I find mathematics useful for

- ☐ dealing with day to day routines
- ☐ dealing with calculations and estimations
- ☐ developing intellectual skills
- ☐ dealing with a variety of work related tasks
- ☐ doing activities such as puzzles and games
- ☐ understanding government policies
- ☐ doing art activities such as drawing and design
- ☐ academic and professional development
- ☐ advancing science and technology
- ☐ communicating ideas related to numbers and measurements

FN-Q13: Select the statement that best describes your view of how mathematics came about. Select only one statement.

☐ People found out about mathematics as they tried to know more about their world.

☐ People came up with mathematics to help them look after their everyday needs.

FN-Q 14: We use different mathematical tools for different purposes. For example, we use numbers to label post boxes. We use addition to calculate restaurant bills. List one example of each, where mathematics is used for:

daily tasks	**work related tasks**

SECTION (B)

Provide your response in the indicated space. Where appropriate, indicate the response using √ in the corresponding box.

Year of Birth: ☐

Highest Class Reached: ☐ High School ☐ Other, specify ____________

Program of Study (For example: Business, Health Science): ____________

If you have held a job before, please complete the following section.

Current Job: ____________

Previous Employment: ____________

Additional comments:

APPENDIX D

Research Questions and Corresponding Questionnaire Items

Research Question	Corresponding EX-Q item(s)	Corresponding IN-Q item(s)	Corresponding FN-Q item(s)
Question 1: What are student conceptions of mathematics as a discipline?	EX-Q1: If we were to describe language, we could say that it is a tool of communication, a means of expressing ourselves, etc. If you were to describe mathematics, what would you say?	IN-Q2: Select the statement that best describes your views of mathematics. Select only one statement. Mathematics is ….	FN-Q7: Select the statement that best describes your view of mathematics and its applications. Select only one statement. I think of mathematics as ….
			FN-Q10: Select the statement that best describes your view of mathematics and its applications. Select only one statement. I think that mathematics is a tool for …
	EX-Q7: Complete the following statement as in the example: To me language is like a key. It opens the door of communication. To me mathematics is like a …	IN-Q3: Select the statement that best describe what mathematics is to you. Complete the statement with a suitable word or phrase. To me mathematics is ….	FN-Q4: Complete the statement with a phrase that expresses how you think of mathematics To me mathematics is like a(n) …
		IN-Q1: Select the statement that best describes your view of how mathematics developed.	FN-Q6: Select the statement that best describes your view of how mathematics developed? Select only one statement.
			FN-Q13: Select the statement that best describes you view of how mathematics came about. Select only one statement.
		IN-Q 4: For each statement, select the choice that reflects your agreement or disagreement with the given statement. Select only one choice.	

Research Question	Corresponding EX-Q item(s)	Corresponding IN-Q item(s)	Corresponding FN-Q item(s)
Question 2: What are student conceptions of the utility of mathematics in everyday life?	EX-Q2: We find language useful because it helps us know and understand others better. It also helps us express our thoughts. Do you find mathematics useful? Why?	IN-Q5: Select the statement(s) that best describe your views about the utility of mathematics You may select more than one statement. I find mathematics useful for …	FN-Q12: Select the statement that best describes your view on the usefulness of mathematics. Select only one statement. I find mathematics useful for…
			FN-Q1: Select the statement that best describes how mathematics helps you. Select only one statement. I find mathematics helpful in …
	EX-Q3: In language, we study about sentence structure because it is useful for forming sentences, about adjectives, because they are useful for describing things, etc. Can you think of lessons that you found useful in mathematics. If so, what were those lessons about?	IN-Q6: Arrange the mathematical subjects in order of importance. The subject that you think is most useful should be ranked as (1).	FN-Q3: Select the mathematical subject that you use most. Select only one subject.
			FN-Q8: Select the mathematical topic that you use most. Select only one topic. (specific topics are listed, e.g. solving equations)
		IN-Q7: Arrange the following lessons in order of importance. The lesson that you think is most useful should be ranked as (1).	FN-Q5: Select the mathematical topic that you think is most useful. You may select more than one topic. (checklist of topics included)
	EX-Q4: Language helps us develop communication skills such as reading, listening, and speaking. Did mathematics help you develop any skills? If so, what are those skills?	IN-Q8: Select the type of skills that mathematics helped you develop. Then, select the specific skills. You may select more than one type of specific skill.	FN-Q9: Select the most useful skill that mathematics helps you develop. Select only one skill.
			FN-Q2: Select the most useful skill that mathematics helps you develop. Select only one skill. (examples of skills listed, e.g. learning and understanding)
			FN-Q11: Select the most useful skill that mathematics helps you develop. You may select more than one skill. (checklist of skills provided)

Research Question	Corresponding EX-Q item(s)	Corresponding IN-Q item(s)	Corresponding FN-Q item(s)
Question 3: What are student conceptions of the link between mathematical knowledge and its applications in the workplace environment?	EX-Q5: Educators use number operations, averages and percentages to come up with student grades. In your own profession, do you use mathematics? If so, could you describe how you use it?	IN-Q9: Can you think of any other professions were mathematics is needed and used? In what ways?	FN-Q14: We use different mathematical tools for different purposes. For example, we use numbers to label post boxes. We use addition to calculate restaurant bills List one example of each, were mathematics is used for : daily tasks work related tasks.
	EX-Q6: Choose a profession, other than your own, where you think that mathematics is used? Could you describe how mathematics is used in this profession?		

General	EX-Q8: Is there any question that you think we should have asked to get to know more about your views of mathematics?	IN-Q10: Are there any views that you would like to express regarding the role of mathematics in our everyday lives and professions.	Additional Comments

APPENDIX E

Categories of Conceptions – Items Ex-Q1 and Ex-Q2

	Categories	Description	Frequency
EX-Q1A	Numbers & Rules	Math is a group of numbers and rules for doing arithmetic problems and calculations.	31%
	Intellectual Exercise	Math is a mental exercise that helps develop intellectual skills, including problem solving.	21%
	Life Line	Math is useful in everyday and professional life.	18%
	School Subject	Math is an essential subject useful in school for future studies.	14%
	Symbolic Language	Math is a symbolic language for expressing relations between shapes and measures.	10%
	Language of Science	Math is the language of science through which the physical world can be described.	5%
	Self Expression	Math is a form of ~~self expression~~ *communication*	1%
EX-Q1B	Calculation Tool	Mathematics is ~~a means of doing~~ *used to do* calculations in a variety of contexts.	34%
	Practical Tool	Mathematics ~~provides the means~~ *i s used* to deal with a variety of everyday life situations including work related ones.	24%
	Intellectual Tool	Math is ~~a means to~~ *used to* develop and enhance intellectual abilities.	21%
	Language Tool	Math is ~~a means of~~ *used for* communication and expression. It provides mathematicians, scientists, accountants, business people and others with the means of communicating with each other through mathematical language.	12%
	Problem Solving Tool	Math is ~~a means of developing~~ *used to develop* problem solving techniques.	7%
	Developmental Tool	Math ~~provides the means~~ *is used for* scientific and technological advancements.	3%

	Categories	Description	Frequency
EX-Q2	Everyday Life Situations	Math provides the means ~~for dealing~~ *to deal with* a variety of everyday life situations.	25.0%
	Calculation & Estimation	Math provides the means ~~for estimating~~ *to estimate* values and performing perform calculations.	25.0%
	Intellectual Development	Math provides the means for intellectual development.	15.0%
	Work Related Situations	Math provides the means ~~for dealing~~ *to deal* with a variety of work related situations.	14%
	Quantities & Measures	~~Math provides the means for representing numerical quantities and measures, as well as ordering and comparing them.~~ *Math provides the means to count and measure, as well as to order and compare.* .	9%
	Academic & Professional Development	Mathematical concepts and ideas are used in several domains. Thus math provides the means for individual academic and professional development.	5%
	Statistics	Math provides the means to organize and present quantitative data.	3%
	Scientific & Technological Innovations	Math provides the means ~~for advancing~~ *to advance* science and technology.	3%
	Communication	Math provides the means ~~of communication~~ *to communicate* between people of different professions.	3%

Notes. The words in strikethrough and italics are used to indicate the modifications to the categories where, where italics indicate the new additions.
Frequencies are arranged in descending order and are rounded to the nearest whole number.

APPENDIX F

Categories of Conceptions – Items EX-Q3 to EX-Q7

Categories of EX-Q3	Frequency	Subcategories (topics)	Frequency for each topic
Arithmetic	44%	Operations and Calculations Percentage and Applications Fractions Number and Place Value Ratio General Multiples Multiplication Tables Estimation Decimals Simplification Problems	60% 12% 9% 8% 3% 3% 1% 1% 1% 1% 1% 1%
Algebra	23%	General Equations Variables and Symbols Formulas Operations Roots	49% 30% 15% 3% 2% 2%
Geometry	12%	General Measurement Volume Areas Shapes Space Geometry Distance Mass Capacity	38% 12% 12% 9% 6% 6% 6% 6% 6%
Statistics	11%	General Graphs Probability Averages F & Z Scores	77% 10% 7% 3% 3%
General Math	5%	no specified topics	-
Calculus	2%	Derivatives Integrals	50% 50%
Foundations	2%	no specified topics	-
Business Math	1%	General Break Even Analysis Supply and Demand	50% 25% 25%

Notes. Frequencies are rounded to the nearest whole number, and arranged in descending order.

Categories of EX-Q4	Frequency	Subcategories (types of skills)	Frequency for each type of skill
Intellectual Skills	48%	Thinking Skills Problem Solving Skills Attention & Concentration Memory Skills Learning & Understanding Intelligence Accuracy & Precision Planning & Organization Imagination Patience & Determination Reviewing & Repeating Challenge & Enjoyment	39% 17% 12% 8% 7% 6% 3% 3% 2% 1% 1% 1%
Mathematical Skills	30%	Arithmetic skills such as dealing with percents. Algebra Skills such as solving equations Statistical Skills such as using charts Geometry Skills such as measurement	66% 13% 10% 10%
Academic, Professional and Everyday Life Skills	17%	Accounting and Bookkeeping Academic such as learning sciences Everyday life skills Banking and Finance Business, Commerce and Economics Investments in Stock Markets Health Care such as dosage administration, Professional such as calculating grades Calculator & Computer skills Engineering	28% 14% 11% 11% 11% 6% 6% 6% 6% 3%
Communication Skills	5%	Language skills Communicating in numbers Dealings between individuals in markets	42% 33% 25%

Notes. Frequencies are rounded to the nearest whole number, and arranged in descending order.

Categories of EX-Q5	Frequency	Subcategories	Subcategory Frequency
Academic Sector	28%	For calculating average and percentage grades	63%
		For teaching math as a school subject	23%
		For schedules and records of absence	14%
Everyday Use	25%	For calculations, percentages, averages	48%
		For determining age, time, personal accounts	35%
		For shopping and determining discounts	13%
		For developing skills such as finding solutions	5%
Health Care Sector	20%	For medication and infusion dosages	50%
		For measuring vital signs such as heart rate	25%
		For physical assessment such as height	9%
		For converting medical rates such as pulse rate	9%
		For medical lab analysis	6%
Business and Finance Sectors	17%	For business transactions	54%
		For inventory control	19%
		For employees	15%
		For banking operations	12%
Professional Sector	10%	For scientific and mathematical calculations such as: speed of sound and light, areas for licensing commercial sites	80%
		For writing numerical and statistical reports	20%

Notes. Frequencies are rounded to the nearest whole number, and arranged in descending order.

Categories of EX-Q6	Frequency	Subcategories	Subcategory Frequency
Business & Trade	46%	For general use in accounting For recording business transactions For administrating financial affaires For general business and trade use For inventory control For organizing work and production schedules For reading & interpreting statistics, For managing employee records	30% 25% 18% 13% 8% 3% 2% 2%
Engineering	20%	Use math theories and formulas for geometric and business calculations. General Use in Engineering Use scientific calculations such as velocity	59% 35% 7%
Money & Banking	17%	Use calculations for dealing with daily transactions and accounts Use topics such as percentages for calculating loans, charges and interests Using math for administration and management purposes such as keeping records Use math for calculation of money exchange rates and amounts	68% 16% 12% 4%
Health Sciences	7%	for dosage calculation and administration for measuring vital signs such as breathing for general use for working with statistics for financial affairs	40% 20% 20% 10% 10%
Academic	5%	for calculation of grades and percentages for teaching as a school subject For administrative purposes such as records of absence, schedules, library loans for studies in sciences, engineering, astronomy	38% 25% 25% 13%
Sciences and Humanities	3%	for scientific calculations in Space Science for social science calculations such as determining areas and population	60% 40%
Other Professions	3%		-

Notes. Frequencies are rounded to the nearest whole number, and arranged in descending order.

Categories of EX-Q7	Frequency	Subcategories	Subcategory Frequency
Symbol and Number	19%	Numbers Symbols Equations Operations Signs Codes Variables	46% 21% 11% 11% 4% 4% 4%
Key	17%	To Facilitate Everyday life and Work Dealings To Intellectual Development To Academic Development To the Future	36% 28% 28% 8%
Thing or Object	14%	Something Complex thing Nothing Skills Challenge Complex Safe Tree, Strange World Corner Stone Moon Life Vein Treasure Pills Trade Theory Questions and Answers	10% 10% 10% 10% 5% 5% 5% 5% 5% 5% 5% 5% 5% 5% 5% 5% 5%
Subject	13%	Useful or essential subject Little or no value subject Complex subject	55% 25% 20%
Means	9%	To develop intellectual skills For dealing with everyday life or work related situations of communication	39% 39% 23%
Tool	8%	Academic Practical for Everyday life and Work Intellectual Communication Tool	33% 33% 17% 9% 9%
Entrance	7%	Door Lock Gate Bridge Entrance	50% 20% 10% 10% 10%
Activity	7%	Problems Exercises Puzzles Mysteries Activities	40% 20% 20% 10% 10%
Language	5%	Foreign or special Language of Numbers and Symbols for Communication and Dealings	43% 29% 29%
Calculations and Solutions	4%	Calculations Solutions	67% 33%

Notes. Frequencies are rounded to the nearest whole number, and arranged in descending order.

APPENDIX G

Breakdown of Frequencies for Multiple Choice Pairs of FN-Q by Age Group, Academic Level and Work Experience

FN-Q7 and matching item FN-Q10

Age Groups	16-19	20-29	30+	Sample
Mathematics is a subject used in everyday life and at work	30%	43%	36%	37%
Mathematics is a tool for dealing with day to day and work related situations	**32%**	**43%**	**36%**	**38%**
Mathematics is a mental exercise that helps develop intellectual abilities	23%	13%	18%	18%
Mathematics is a tool for developing our ability to think	**41%**	**26%**	**36%**	**32%**
Mathematics is a mental activity used to develop problem solving techniques	15%	11%	18%	13%
Mathematics is a tool for developing problem solving techniques	**7%**	**3%**	**11%**	**5%**
Mathematics is a group of numbers and rules for doing calculations	11%	10%	7%	10%
Mathematics is a tool for dealing with numbers	**10%**	**11%**	**11%**	**11%**
Mathematics is a school subject used in learning other subjects and for future studies	9%	9%	14%	9%
Mathematics a tool for studying other subjects	**1%**	**3%**	**4%**	**3%**
Mathematics is a symbolic language that expresses relations between objects and quantities	4%	6%	7%	5%
Mathematics is a tool for representing relations between objects and quantities	**1%**	**2%**	**0%**	**2%**
Mathematics is an activity that allows us to express our creativity	6%	4%	0%	4%
Mathematics is a tool for enhancing innovations	**5%**	**7%**	**4%**	**5%**
Mathematics is a language of science used to describe the physical world	1%	2%	0%	1%
Mathematics is a tool for explaining things such as rules of gravity	**1%**	**1%**	**0%**	**1%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.

Academic Level	<HS	HS	>HS	Sample
Mathematics is a subject used in everyday life and at work	50%	36%	21%	37%
Mathematics is a tool for dealing with day to day and work related situations	**63%**	**36%**	**14%**	**38%**
Mathematics is a mental exercise that helps develop intellectual abilities	13%	18%	36%	18%
Mathematics is a tool for developing our ability to think	**23%**	**32%**	**57%**	**32%**
Mathematics is a mental activity used to develop problem solving techniques	10%	13%	21%	13%
Mathematics is a tool for developing problem solving techniques	**0%**	**6%**	**14%**	**5%**
Mathematics is a group of numbers and rules for doing calculations	10%	10%	7%	10%
Mathematics is a tool for dealing with numbers	**5%**	**13%**	**7%**	**11%**
Mathematics is a school subject used in learning other subjects and for future studies	8%	10%	0%	9%
Mathematics a tool for studying other subjects	**0%**	**2%**	**0%**	**3%**
Mathematics is a symbolic language that expresses relations between objects and quantities	10%	5%	0%	5%
Mathematics is a tool for representing relations between objects and quantities	**3%**	**2%**	**0%**	**2%**
Mathematics is an activity that allows us to express our creativity	0%	5%	7%	4%
Mathematics is a tool for enhancing innovations	**0%**	**7%**	**7%**	**5%**
Mathematics is a language of science used to describe the physical world	0%	2%	0%	1%
Mathematics is a tool for explaining things such as rules of gravity	**3%**	**1%**	**0%**	**1%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.

Work Experience	Exp.	No Exp.	Sample
Mathematics is a subject used in everyday life and at work	38%	36%	37%
Mathematics is a tool for dealing with day to day and work related situations	**38%**	**38%**	**38%**
Mathematics is a mental exercise that helps develop intellectual abilities	15%	19%	18%
Mathematics is a tool for developing our ability to think	**32%**	**31%**	**32%**
Mathematics is a mental activity used to develop problem solving techniques	19%	12%	13%
Mathematics is a tool for developing problem solving techniques	**9%**	**5%**	**5%**
Mathematics is a group of numbers and rules for doing calculations	11%	10%	10%
Mathematics is a tool for dealing with numbers	**13%**	**10%**	**11%**
Mathematics is a school subject used in learning other subjects and for future studies	9%	9%	9%
Mathematics a tool for studying other subjects	**2%**	**3%**	**3%**
Mathematics is a symbolic language that expresses relations between objects and quantities	6%	5%	5%
Mathematics is a tool for representing relations between objects and quantities	**0%**	**3%**	**2%**
Mathematics is an activity that allows us to express our creativity	2%	5%	4%
Mathematics is a tool for enhancing innovations	**4%**	**6%**	**5%**
Mathematics is a language of science used to describe the physical world	0%	2%	1%
Mathematics is a tool for explaining things such as rules of gravity	**0%**	**2%**	**1%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.

FN-Q6 and matching item FN-Q13

Age Groups	16-19	20-29	30+	Sample
People discovered mathematics as they attempted to understand the world around them.	60%	65%	68%	64%
People found out about mathematics as they tried to know more about their world.	**49%**	**54%**	**36%**	**49%**
People created mathematics to address their practical needs.	40%	35%	32%	35%
People came up with mathematics to help them look after their everyday needs.	**48%**	**46%**	**50%**	**48%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.

Academic Level	<HS	HS	>HS	Sample
People discovered mathematics as they attempted to understand the world around them.	68%	63%	79%	64%
People found out about mathematics as they tried to know more about their world.	**35%**	**51%**	**71%**	**49%**
People created mathematics to address their practical needs.	33%	37%	21%	35%
People came up with mathematics to help them look after their everyday needs.	**58%**	**48%**	**21%**	**48%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.

Work Experience	Exp.	No Exp.	Sample
People discovered mathematics as they attempted to understand the world around them.	70%	62%	64%
People found out about mathematics as they tried to know more about their world.	**49%**	**49%**	**49%**
People created mathematics to address their practical needs.	30%	37%	35%
People came up with mathematics to help them look after their everyday needs.	**43%**	**49%**	**48%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.

FN-Q12 and matching item FN-Q1

Age Groups	16-19	20-29	30+	Sample
Mathematics is useful for dealing with day to day routines	23%	33%	29%	28%
Mathematics is helpful in telling the time	**12%**	**4%**	**7%**	**8%**
Mathematics is useful for dealing with calculations and estimations	17%	21%	18%	19%
Mathematics is helpful in finding out how much money is left after shopping	**22%**	**14%**	**21%**	**17%**
Mathematics is useful for developing intellectual skills	17%	11%	18%	14%
Mathematics is helpful in dealing with difficult situations that require analysis	**27%**	**35%**	**54%**	**34%**
Mathematics is useful for communicating ideas related to numbers and measurements	9%	11%	18%	12%
Mathematics is helpful in describing distances between places	**2%**	**3%**	**4%**	**3%**
Mathematics is useful for advancing science and technology	10%	9%	4%	8%
Mathematics is helpful in finding new ways to improve our world	**22%**	**25%**	**14%**	**24%**
Mathematics is useful for doing activities such as puzzles or games	9%	4%	0%	5%
Mathematics is helpful in solving puzzles in magazines or newspapers	**7%**	**2%**	**0%**	**3%**
Mathematics is useful for dealing with a variety of work related tasks	7%	4%	0%	5%
Mathematics is helpful in reading task and duties schedule	**0%**	**4%**	**0%**	**2%**
Mathematics is useful for doing art activities such as drawing and design	1%	2%	4%	3%
Mathematics is helpful in designing patters for weaving	**0%**	**0%**	**0%**	**0%**
Mathematics is useful for academic and professional development	2%	1%	4%	2%
Mathematics is helpful in taking courses to improve some computer skills	**2%**	**6%**	**0%**	**4%**
Mathematics is useful for understanding government policies	0%	0%	0%	0%
Mathematics is helpful in understanding how the social security system works	**0%**	**3%**	**0%**	**1%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.

Academic Level	<HS	HS	>HS	Sample
Mathematics is useful for dealing with day to day routines **Mathematics is helpful in telling the time**	35% **10%**	27% **7%**	21% **14%**	28% **8%**
Mathematics is useful for dealing with calculations and estimations **Mathematics is helpful in finding out how much money is left after shopping**	18% **25%**	21% **17%**	7% **0%**	19% **17%**
Mathematics is useful for developing intellectual skills **Mathematics is helpful in dealing with difficult situations that require analysis**	15% **25%**	13% **35%**	36% **50%**	14% **34%**
Mathematics is useful for communicating ideas related to numbers and measurements **Mathematics is helpful in describing distances between places**	18% **3%**	10% **2%**	14% **7%**	12% **3%**
Mathematics is useful for advancing science and technology **Mathematics is helpful in finding new ways to improve our world**	3% **23%**	10% **23%**	7% **29%**	8% **24%**
Mathematics is useful for doing activities such as puzzles or games **Mathematics is helpful in solving puzzles in magazines or newspapers**	3% **0%**	6% **5%**	7% **0%**	5% **3%**
Mathematics is useful for dealing with a variety of work related tasks **Mathematics is helpful in reading task and duties schedule**	5% **3%**	5% **2%**	0% **0%**	5% **2%**
Mathematics is useful for doing art activities such as drawing and design **Mathematics is helpful in designing patters for weaving**	3% **0%**	2% **0%**	0% **0%**	3% **0%**
Mathematics is useful for academic and professional development **Mathematics is helpful in taking courses to improve some computer skills**	3% **8%**	2% **3%**	0% **0%**	2% **4%**
Mathematics is useful for understanding government policies **Mathematics is helpful in understanding how the social security system works**	0% **0%**	0% **2%**	0% **0%**	0% **1%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.

Work Experience	Exp.	No Exp.	Sample
Mathematics is useful for dealing with day to day routines	30%	28%	28%
Mathematics is helpful in telling the time	**13%**	**7%**	**8%**
Mathematics is useful for dealing with calculations and estimations	23%	28%	19%
Mathematics is helpful in finding out how much money is left after shopping	**19%**	**17%**	**17%**
Mathematics is useful for developing intellectual skills	11%	15%	14%
Mathematics is helpful in dealing with difficult situations that require analysis	**40%**	**32%**	**34%**
Mathematics is useful for communicating ideas related to numbers and measurements	11%	12%	12%
Mathematics is helpful in describing distances between places	**2%**	**3%**	**3%**
Mathematics is useful for advancing science and technology	6%	9%	8%
Mathematics is helpful in finding new ways to improve our world	**17%**	**25%**	**24%**
Mathematics is useful for doing activities such as puzzles or games	6%	5%	5%
Mathematics is helpful in solving puzzles in magazines or newspapers	**2%**	**5%**	**3%**
Mathematics is useful for dealing with a variety of work related tasks	2%	5%	5%
Mathematics is helpful in reading task and duties schedule	**2%**	**2%**	**2%**
Mathematics is useful for doing art activities such as drawing and design	4%	2%	3%
Mathematics is helpful in designing patters for weaving	**0%**	**0%**	**0%**
Mathematics is useful for academic and professional development	0%	2%	2%
Mathematics is helpful in taking courses to improve some computer skills	**8%**	**3%**	**4%**
Mathematics is useful for understanding government policies	0%	0%	0%
Mathematics is helpful in understanding how the social security system works	**0%**	**2%**	**1%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.

FN-Q3 and matching item FN-Q8

Age Groups	16-19	20-29	30+	Sample
Arithmetic is the most useful subject	72%	89%	93%	82%
Numbers and calculations are the most useful topic	**89%**	**90%**	**86%**	**88%**
Algebra is the most useful subject	16%	3%	4%	8%
Solving equations is the most useful topic	**9%**	**8%**	**11%**	**8%**
Statistics is the most useful subject	9%	7%	0%	7%
Averages and probabilities are the most useful topic	**1%**	**2%**	**4%**	**3%**
Geometry is the most useful subject	2%	2%	4%	3%
Shapes and measurement are the most useful topic	**0%**	**0%**	**0%**	**0%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.

Academic Level	<HS	HS	>HS	Sample
Arithmetic is the most useful subject	80%	82%	93%	82%
Numbers and calculations are the most useful topic	**85%**	**90%**	**93%**	**88%**
Algebra is the most useful subject	0%	9%	7%	8%
Solving equations is the most useful topic	**8%**	**9%**	**0%**	**8%**
Statistics is the most useful subject	13%	6%	0%	7%
Averages and probabilities are the most useful topic	**3%**	**1%**	**7%**	**3%**
Geometry is the most useful subject	5%	2%	0%	3%
Shapes and measurement are the most useful topic	**0%**	**0%**	**0%**	**0%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.

Work Experience	Exp.	No Exp.	Sample
Arithmetic is the most useful subject	91%	79%	82%
Numbers and calculations are the most useful topic	**85%**	**89%**	**88%**
Algebra is the most useful subject	2%	9%	8%
Solving equations is the most useful topic	**95**	**8%**	**8%**
Statistics is the most useful subject	4%	8%	7%
Averages and probabilities are the most useful topic	**4%**	**2%**	**3%**
Geometry is the most useful subject	0%	4%	3%
Shapes and measurement are the most useful topic	**0%**	**0%**	**0%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.

FN-Q9 and matching item FN-Q2

Age Groups	16-19	20-29	30+	Sample
Intellectual Skills	69%	54%	46%	58%
Learning and understanding	**42%**	**55%**	**71%**	**50%**
Practical Skills	14%	25%	29%	23%
Knowing the time difference between countries	**16%**	**2%**	**0%**	**7%**
Mathematical Skills	7%	10%	18%	10%
Solving equations	**27%**	**24%**	**11%**	**25%**
Communication Skills	9%	9%	4%	8%
Communicating using numbers	**14%**	**18%**	**18%**	**16%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.

Academic Level	<HS	HS	>HS	Sample
Intellectual Skills	53%	59%	64%	58%
Learning and understanding	**53%**	**52%**	**43%**	**50%**
Practical Skills	25%	23%	0%	23%
Knowing the time difference between countries	**8%**	**7%**	**0%**	**7%**
Mathematical Skills	5%	10%	21%	10%
Solving equations	**23%**	**23%**	**36%**	**25%**
Communication Skills	15%	7%	7%	8%
Communicating using numbers	**15%**	**17%**	**21%**	**16%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.

Work Experience	Exp.	No. Exp	Sample
Intellectual Skills	55%	58%	58%
Learning and understanding	**62%**	**47%**	**50%**
Practical Skills	21%	23%	23%
Knowing the time difference between countries	**4%**	**7%**	**7%**
Mathematical Skills	13%	9%	10%
Solving equations	**19%**	**27%**	**25%**
Communication Skills	6%	8%	8%
Communicating using numbers	**13%**	**17%**	**16%**

Notes. The text written in bold refers to the original item and the text in regular font refers to the matching form (matching item) for each multiple choice pair.
The percentages indicate the frequency of responses rounded to the nearest whole number and arranged in descending order.